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# Performance analysis of noncoherent differential phase shift keying using Post-Detection Selection Combining over a Rayleigh fading channel

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## Monterey, California

# THESIS

**PERFORMANCE ANALYSIS OF NONCOHERENT DIFFERENTIAL PHASE SHIFT KEYING USING POST-DETECTION SELECTION COMBINING OVER A RAYLEIGH FADING CHANNEL**

by

Tahir Conka

June 1998

Thesis Advisor: Tri T. Ha
Co-Advisor: Ralph D. Hippenstiel

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Numerical results obtained for Post-Detection Selection Combining are compared to Selection Combining and Equal Gain Combining. The Post-Detection Selection Combining method is shown to be superior to the Selection Combining method but inferior to Equal Gain Combining method for a non-coherent DPSK receiver operating over a Rayleigh fading channel.

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# PERFORMANCE ANALYSIS OF NONCOHERENT DIFFERENTIAL PHASE SHIFT KEYING USING POST-DETECTION SELECTION COMBINING OVER A RAYLEIGH FADING CHANNEL

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# ABSTRACT

In this thesis, the performance analysis of a noncoherent Differential Phase Shift Keying (DPSK) receiver using Post-Detection Selection Combining techniques over a Rayleigh fading channel is investigated.

Post-Detection Selection Combining (PDSC) is evaluated and compared to Equal Gain Combining (EGC) and Selection Combining (SC), the two common diversity techniques discussed in the literature.

Numerical results obtained for Post-Detection Selection Combining are compared to Selection Combining and Equal Gain Combining. The Post-Detection Selection Combining method is shown to be superior to the Selection Combining method but inferior to Equal Gain Combining method for a non-coherent DPSK receiver operating over a Rayleigh fading channel.

# TABLE OF CONTENTS

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This thesis is dedicated to my wife, Yasemin, my parents, whose guidance and encouragement have inspired me, to make them proud, and my son Sarp, who brings indescribable joy into my life.

# I. INTRODUCTION

## A. BACKGROUND

The use of a natural medium for communications systems, especially for wireless and mobile radio communications systems, implies unavoidable degradation associated with the randomness that accompanies natural phenomena. Refraction, reflection, diffraction, scattering, focusing attenuation and other factors cause variations between the received signal level and a calculated free-space level for a given transmitter output. The degradation is usually due to natural causes but can also be caused by man made items such as huge buildings in an urban area [1,2].

Although only the direct wave is desired, several distinguishable paths between the transmitter and receiver do exist. These are typically due to reflections. These paths cause several waves to arrive at the receiver at slightly different times and produce fading. The interference between a direct wave and a reflected wave is called multi-path fading [2].

The electro-magnetic waves tend to arrive at the receiver over different paths having different propagation delays. The received signal consists of components having randomly distributed amplitudes, phases and angles of arrival. When there is no single line-of-sight path between the transmitter and the receiver, the received signal is modeled as a Rayleigh random variable. Its probability density function (pdf) is given by [4]

$$f(z)=\begin{cases}\dfrac{z}{\sigma^2}\exp\left(-\dfrac{z}{2\sigma^2}\right) & z\geq 0\\ 0 & otherwise\end{cases}, \qquad (1)$$

where $2\sigma^2$ is the average power of faded signal, and z is the amplitude of the received signal.

The resulting communication channel is called a Rayleigh fading channel. When the line-of-sight component is not zero, the received signal is modeled as a Ricean random variable. Its pdf is given by [4]

$$f(z)=\begin{cases}\dfrac{z}{\sigma^2}\exp\left(-\dfrac{z^2+a^2}{2\sigma^2}\right)I_0\left(\dfrac{a\,z}{\sigma^2}\right) & a\geq 0, z\geq 0\\ 0 & otherwise\end{cases}\quad, \tag{2}$$

where $a^2$ is the average power of the unfaded (direct) signal component, $2\sigma^2$ is the average power of the faded signal component and $I_0(\bullet)$ is the zeroth-order modified Bessel function of the first kind. This resulting communication channel is called a Ricean fading channel [2,3].

Rayleigh fading severely degrades the average probability of the bit error performance of the receiver. One way of minimizing this problem is to increase the transmitter power or the antenna size. However, such a solution requires costly engineering alterations making other alternatives very attractive. It is well known, that diversity combining is one of the most effective alternatives [1].

To reduce the effects of fading, one of several diversity techniques can be implemented. Diversity is a procedure to receive or transmit the same symbol multiple times in order to provide redundancy at the receiver. The basic idea of diversity is that some of the received redundant symbols will be more reliable than the others, and the demodulation decision will be made using the more reliable symbols. In order to be useful, each redundant symbol must be received independently [4]. The diversity can be implemented as space, time, frequency, angle and polarization diversity. Space diversity consists of using multiple antennas at the receiver in order to receive the transmitted symbol multiple times via multiple paths. Time diversity is employed by transmitting the

same symbol multiple times. In frequency diversity, the symbol is transmitted on multiple carrier frequencies at the same time. Angle diversity is implemented by a set of directive antennas where each one responds independently to a wave to produce an uncorrelated faded signal. Polarization diversity is possible with two orthogonal polarizations. Combining the signals from the diversity branches allows a significant performance improvement. Diversity combining can be accomplished prior to or after detection, which are called pre- and post-detection, respectively. Diversity combining can be linear or non-linear. In this thesis, only linear combining methods are analyzed. Unlike the other methods used for reducing fading effects, diversity combining lowers costs and reduces complexity in the receiver [2]. In this thesis, a new method referred to as Post-Detection Selection Combining (PDSC) is presented and compared with Selection Combining and Equal Gain Combining [1,3].

Post-Detection Selection Combining (PDSC) is invented by Professors Tri T. Ha and Ralph D. Hippenstiel [14]. This method selects the signal with the largest amplitude or combines the two (three) received signals with the two (three) largest amplitudes after detection. Since combining is done after detection, the new method is named as First, Second, and Third Order Post-Detection Selection Combining and is denoted by PDSC-1, PDSC-2 and PDSC-3, respectively.

## B. SYSTEM DESCRIPTION

A noncoherent differential phase shift keyed (DPSK) receiver, as shown in Fig. 1, is used to demonstrate the performance of Post-Detection Selection Combining. As a noncoherent receiver, the DPSK receiver requires no phase recovery and can be build inexpensively. For this reason, it is widely used in wireless communications systems.

While DPSK signaling has the advantage of reduced receiver complexity, its energy efficiency is inferior to that of coherent PSK by about 3 dB [4]. The average probability of error for DPSK in additive white Gaussian noise is given by [4]

$$P_e = \frac{1}{2}\exp\left(\frac{-E_b}{N_0}\right), \tag{3}$$

where $E_b / N_0$ is the bit-energy-to-noise-density ratio.

Before we analyze the Post-Detection Selection Combining, we need to consider general structure of the DPSK receiver. The phase difference between two successive bits is used to convey information. Whenever the data bit $b_i = 0$ is transmitted, the waveform for two consecutive differentially encoded bits $c_{i-1}c_i$ is given by [6]

$$\begin{aligned} v^{(1)}(t) &= \pm A\, p_T(t-(i-1)T)\cos(2\pi f_c t+\theta) \pm A\, p_T(t-iT)\cos(2\pi f_c t+\theta) \\ &= [\pm A\, p_T(t-(i-1)T) \pm A\, p_T(t-iT)]\cos(2\pi f_c t+\theta). \end{aligned} \tag{4}$$

When the data bit $b_i = 1$ is transmitted, the waveform, which has a phase change of $\pi$ radians between two consecutive bits, is given by

$$\begin{aligned} v^{(2)}(t) &= \pm A\, p_T(t-(i-1)T)\cos(2\pi f_c t+\theta) \mp A\, p_T(t-iT)\cos(2\pi f_c t+\theta) \\ &= [\pm A\, p_T(t-(i-1)T) \mp A\, p_T(t-iT)]\cos(2\pi f_c t+\theta), \end{aligned} \tag{5}$$

where $\theta$ is the signal phase, $f_c$ is the carrier frequency, A is the amplitude and the $\pm$ signs denotes the polarity of the two consecutive bits. The pulse $p_T(t-iT)$ is given by

$$p_T(t-iT) = \begin{Bmatrix} 1 , & iT < t \le (i+1)T) \\ 0 , & otherwise \end{Bmatrix}. \tag{6}$$

When $b_1 = 0$ is transmitted, the received signal is of the form

$$r(t)=\left[\pm A\, p_T(t)\ \pm A\, p_T\,(t-T)\right]\cos\left(2\pi\, f_c\, t+\theta\right)+n(t)\,, \tag{7}$$

where n(t) is additive white Gaussian noise with zero mean and power spectral density $\frac{N_0}{2}$.

The in-phase outputs of the receiver are given by

$$Y_{1c}=\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{}'\right)+\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{}''\right)=\pm AT\cos\theta\ +N_{1c}\,, \tag{8}$$

$$Y_{2c}=\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{}'\right)-\left(\pm\frac{AT}{2}\cos\theta\ +N_{1c}{}''\right)=N_{2c}\,,$$

where

$$N_{1c}=N_{1c}{}'+N_{1c}{}''\ ,\quad N_{2c}=N_{1c}{}'-N_{1c}{}''\,, \tag{9}$$

and

$$N_{1c}{}'=\int_0^T n(t)\cos(2\pi\, f_c\, t)dt\,, \tag{10}$$

$$N_{1c}{}''=\int_T^{2T} n(t)\cos(2\pi\, f_c\, t)dt\,.$$

The quadrature outputs of the receiver are obtained as

$$Y_{1s}=\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{}'\right)+\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{}''\right)=\mp AT\sin\theta\ +N_{1s}\,, \tag{11}$$

$$Y_{2s}=\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{}'\right)-\left(\pm\frac{AT}{2}(-\sin\theta\ )+N_{1s}{}''\right)=N_{2s}\,,$$

where

$$N_{1s}=N_{1s}{}'+N_{1s}{}''\ ,\quad N_{2s}=N_{1s}{}''-N_{1s}{}''\,, \tag{12}$$

and

$$N_{1s}{}'=\int_0^T n(t)\sin(2\pi\, f_c\, t)dt\,, \tag{13}$$

$$N_{1s}{}''=\int_T^{2T} n(t)\sin(2\pi\, f_c\, t)dt\,.$$

$N_{1c}$, $N_{2c}$, $N_{1s}$ and $N_{2s}$ are independent, identically distributed (i.i.d.) Gaussian random variables with zero mean and variance equal to $\frac{N_0 T}{2}$.

Assuming that $b_1 = 0$ is transmitted, the outputs of the signal branch are

$$\begin{aligned} Y_{1c} &= \pm AT\cos\theta + N_{1c}, \\ Y_{1s} &= \mp AT\sin\theta + N_{1s}, \end{aligned} \tag{14}$$

and the outputs of the non-signal branch are

$$\begin{aligned} Y_{2c} &= N_{2c}, \\ Y_{2s} &= N_{2s}. \end{aligned} \tag{15}$$

As seen in Fig. 1, the decision variable for the signal branch is given by

$$Y_{1c}^{\ 2} + Y_{1s}^{\ 2} = (\pm AT\cos\theta + N_{1c})^2 + (\mp AT\sin\theta + N_{1s})^2. \tag{16}$$

For the non-signal branch, the corresponding variable is given by

$$Y_{2c}^{\ 2} + Y_{2s}^{\ 2} = N_{2c}^{\ 2} + N_{2s}^{\ 2}. \tag{17}$$

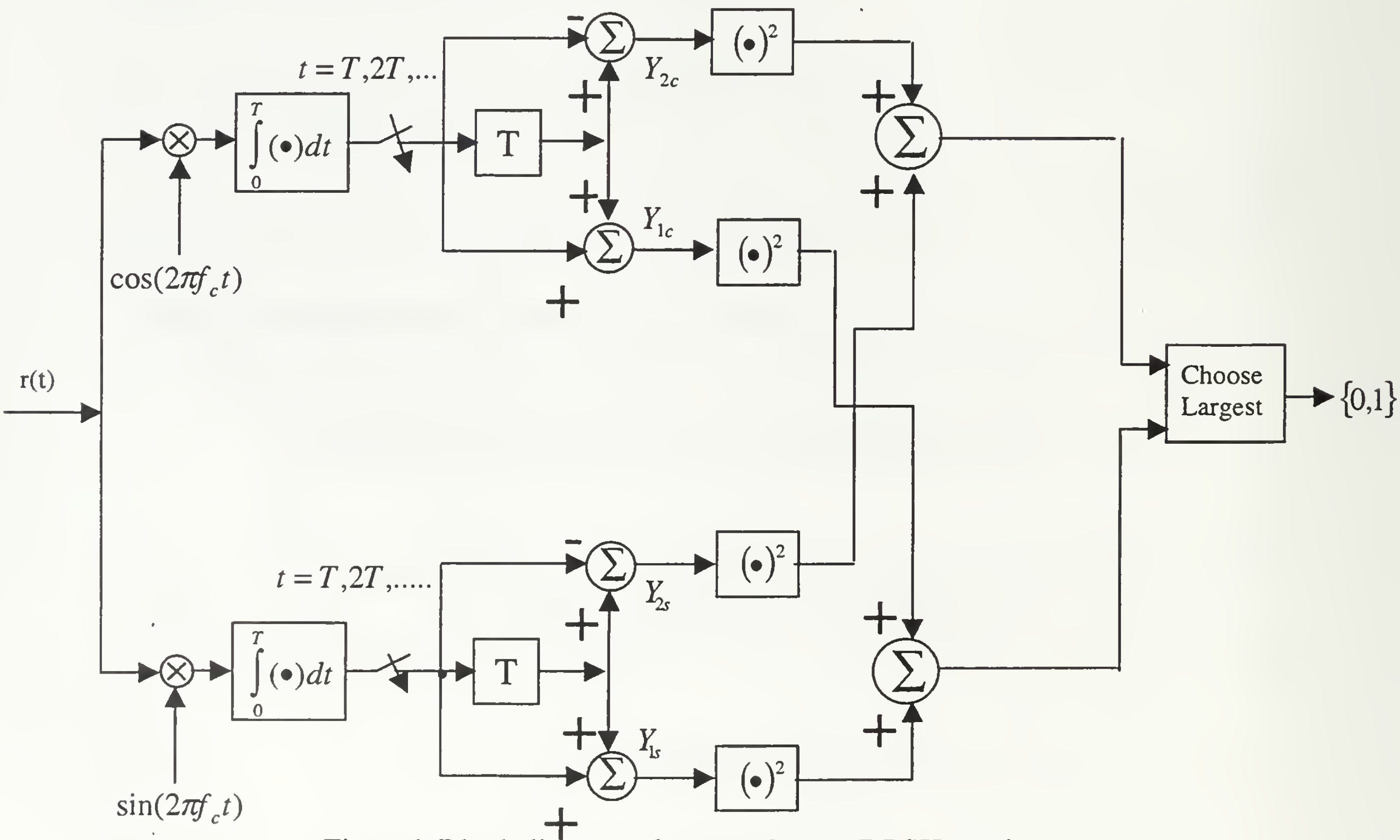


Figure 1. Block diagram of a noncoherent DPSK receiver

For convenience , we normalize (16) as follows:

$$\begin{aligned} Y_{1c}^2+Y_{1s}^2 &= \left[\sqrt{\frac{T}{2}}\left(\pm\sqrt{2T}A\cos\theta+\frac{\sqrt{2}}{\sqrt{T}}N_{1C}\right)\right]^2+\left[\sqrt{\frac{T}{2}}\left(\mp\sqrt{2T}A\sin\theta+\frac{\sqrt{2}}{\sqrt{T}}N_{1S}\right)\right]^2 \\ &= \frac{T}{2}\left(\pm\sqrt{2T}A\cos\theta+\frac{\sqrt{2}}{\sqrt{T}}N_{1C}\right)^2+\frac{T}{2}\left(\mp\sqrt{2T}A\sin\theta+\frac{\sqrt{2}}{\sqrt{T}}N_{1S}\right)^2 \quad . \end{aligned} \tag{18}$$

By setting $Y_1=\frac{2}{T}\left(Y_{1c}^2+Y_{1s}^2\right)$, $n_{1c}=\sqrt{\frac{2}{T}}N_{1C}$ , $n_{1s}=\sqrt{\frac{2}{T}}N_{1s}$ , $A'=\pm\frac{\sqrt{2T}}{2}A$, we have the decision variable

$$Y_1=\left(2A'\cos\theta+n_{1c}\right)^2+\left(-2A'\sin\theta+n_{1s}\right)^2. \tag{19}$$

For a Rayleigh fading channel, $A'$ is a Rayleigh random variable with parameter $\sigma$ as given in (1). Furthermore, for a uniform variable $\theta$, $A'\cos\theta$ and $-A'\sin\theta$ are Gaussian random variables with zero mean and variance $\sigma^2$[11]. The variables $2A'\cos\theta$ and $-2A'\sin\theta$ are both zero mean Gaussian random variables with variance $4\sigma^2$.The zero mean Gaussian random variable $n_{1c}$ and $n_{1s}$ are independent with variance $\sigma_n^2=N_0$. Therefore the random variable $Y_1$ (19) is a central chi-square random variable with the following density function [4]

$$f_{Y_1}(y_1)=\frac{1}{2\sigma_1^2}\exp\left(-\frac{y_1}{2\sigma_1^2}\right), \qquad y_1\ge 0, \tag{20}$$

where

$$\sigma_1^2=4\sigma^2+\sigma_n^2=4\sigma^2+N_0. \tag{21}$$

Define the average energy per diversity channel as

$$\overline{E}=2\sigma^2; \tag{22}$$

then, for L diversity channels, the bit energy is

$$\overline{E_b} = L\overline{E}. \tag{23}$$

For the non-signal branch, the decision variable $Y_2 = \frac{2}{T}\left(Y_{2c}^2 + Y_{2s}^2\right)$ can be written as

$$Y_2 = n_{2c}^2 + n_{2s}^2, \tag{24}$$

where $n_{2c} = \sqrt{\frac{2}{T}}N_{2c}$ and $n_{2s} = \sqrt{\frac{2}{T}}N_{2s}$ are zero mean Gaussian random variables with variance $\sigma_n^2 = N_0$. The density function of $Y_2$ is a central chi-square density function [4]

$$f_{Y_2}(y_2) = \frac{1}{2\sigma_2^2}\exp\left(-\frac{y_2}{2\sigma_2^2}\right), \qquad y_2 \geq 0, \tag{25}$$

where

$$\sigma_2^2 = \sigma_n^2 = N_0. \tag{26}$$

## C. OBJECTIVE

In the following chapters, probability density functions are obtained and the bit error probabilities are derived for PDSC-1, PDSC-2, and PDSC-3. In Chapter VI, Post-Detection Selection Combining, Selection Combining and Equal Gain Combining are compared.

The primary objectives are to

(a) present the new Post-Detection Selection Combining method,

(b) make the necessary bit error rate derivations, and

(c) compare the performances of Post-Detection Selection Combining, Selection Combining and Equal Gain Combining.

# II. REVIEW OF PREVIOUS WORK

## A. EQUAL GAIN COMBINING

The Equal Gain Combining (EGC) method adds in a noncoherent fashion equally weighted branch signals. Although EGC is one of the most commonly used diversity techniques, the receiver is dependent on the order of the diversity [3]. The bit error probability expression for Equal Gain Combining is given by [4]

$$P_b = \sum_{k=0}^{L-1} \binom{L-1+k}{k} \frac{\left(1+\frac{2E_b}{LN_0}\right)^k}{\left(2+\frac{2E_b}{LN_0}\right)^{L+k}} \quad . \tag{27}$$

## B. FIRST ORDER SELECTION COMBINING (SC-1)

In first order selection combining (SC-1) the signal with the largest amplitude (hence the largest signal-to-noise ratio) is selected as shown Fig. 2.

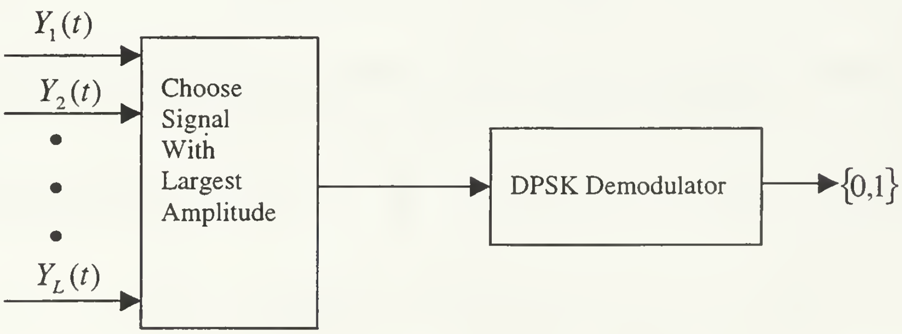


Figure 2. Selection Combining with L-th order Diversity

The conditional bit error probability for SC-1 is simply the bit error probability for DPSK conditional on the signal-to-noise ratio $\gamma$ [4]

$$P_b(\gamma) = \frac{1}{2} e^{-\gamma} . \tag{28}$$

The pdf of $\gamma$ is given by [3]

$$f(\gamma) = L\alpha e^{-\alpha\gamma}\left(1 - e^{-\alpha\gamma}\right)^{L-1}, \tag{29}$$

where

$$\alpha = \frac{N_0}{\overline{E}} . \tag{30}$$

Here $\overline{E}$ is the average energy per diversity bit, and $\overline{E}/N_0$ is the average diversity bit energy-to-noise density ratio.

The bit error probability for SC-1 can be evaluated as follows [3]

$$P_b = \int_0^\infty P_b(\gamma) f(\gamma) d\gamma . \tag{31}$$

Inserting (28) and (29) into (31) and performing the integration, we obtain

$$P_b = \frac{L}{2} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{\alpha}{1+\alpha+k\alpha} . \tag{32}$$

We note that the average bit energy $\overline{E_b}$ is related to the average diversity bit energy $\overline{E}$ by $\overline{E_b} = L\overline{E}$. Thus using the identity $\alpha = \dfrac{LN_0}{\overline{E}_b}$, we obtain a result in terms of $\dfrac{\overline{E}_b}{N_0}$ as

$$P_b = \frac{L}{2} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \frac{\left(\dfrac{LN_0}{\overline{E}_b}\right)}{1 + \left(\dfrac{LN_0}{\overline{E}_b}\right) + k\left(\dfrac{LN_0}{\overline{E}_b}\right)} . \tag{33}$$

## C. SECOND ORDER SELECTION COMBINING (SC-2)

In second order selection combining (SC-2), two signals with the largest amplitudes or signal-to-noise ratios at the input of the DPSK demodulator are combined. The bit error probability conditional on the signal-to-noise ratio $\gamma$ is given by [3]

$$P_b(\gamma)=\frac{1}{8}e^{-\gamma}\left[4+\gamma\right]. \tag{34}$$

The pdf of $\gamma$ is also given in [3]

$$f(\gamma)=L(L-1)\alpha\, e^{-\alpha\gamma}\left[\frac{\alpha\gamma}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left(1-e^{-\frac{\alpha k\gamma}{2}}\right)\right], \tag{35}$$

where

$$\alpha=\frac{N_0}{\overline{E}}, \tag{36}$$

as defined in (30). Substituting (34) and (35) into (31) yields

$$P_b=\int_0^\infty \frac{1}{8}e^{-\gamma}\left[4+\gamma\right]L(L-1)\alpha\; e^{-\alpha\gamma}\left[\frac{\alpha\gamma}{2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left(1-e^{-\frac{\alpha\gamma k}{2}}\right)\right]d\gamma\ . \tag{37}$$

By performing this integration, the bit error probability is found to be [3]

$$\begin{aligned}P_b &=\left(\frac{\alpha}{1+\alpha}\right)^2\frac{L(L-1)}{8}\\ &\times\left\{2+\frac{1}{\alpha+1}+\sum_{k=1}^{L-2}\binom{L-2}{k}(-1)^k\left[\frac{(1+\alpha)\left[4(2+2\alpha+\alpha k)+4\right]+\alpha k}{(2+2\alpha+\alpha k)^2}\right]\right\}.\end{aligned} \tag{38}$$

The bit error probability is obtained in terms of $\dfrac{\overline{E}_b}{N_0}$ as

$$P_b = \left( \frac{\frac{LN_0}{\overline{E}_b}}{1+\frac{LN_0}{\overline{E}_b}} \right)^2 \frac{L(L-1)}{8} \left\{ 2 + \frac{1}{\frac{LN_0}{\overline{E}_b}+1} \right. \tag{39}$$

$$\left. + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k \left[ \frac{\left(1+\frac{LN_0}{\overline{E}_b}\right)\left[4\left(2+2\frac{LN_0}{\overline{E}_b}+\frac{LN_0}{\overline{E}_b}k\right)+4\right]+\frac{LN_0}{\overline{E}_b}k}{\left(2+2\frac{LN_0}{\overline{E}_b}+\frac{LN_0}{\overline{E}_b}k\right)^2} \right] \right\}.$$

## D. THIRD ORDER SELECTION COMBINING (SC-3)

For the third order selection combining (SC-3), three signals having the three largest amplitudes or signal-to-noise ratios at the input of the DPSK receiver are combined and the conditional bit error probability for DPSK is given by [3]

$$P_b(\gamma) = \frac{1}{32} e^{-\gamma} \left[ 16 + 6\gamma + \frac{1}{2}\gamma^2 \right]. \tag{40}$$

The pdf of $\gamma$ for SC-3 is given by [3]

$$f(\gamma) = \frac{L(L-1)(L-2)}{2} \alpha e^{-\alpha \cdot \gamma} \tag{41}$$

$$\times \left\{ \frac{\alpha^2\gamma^2}{6} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[ \alpha\gamma k - 3\left(1-\exp\left(-\frac{k\alpha\gamma}{3}\right)\right)\right]\right\}.$$

Substituting (40) and (41) in (31) yields

$$P_b = \int_0^\infty \frac{1}{32} e^{-\gamma}\left[16 + 6\gamma + \frac{1}{2}\gamma^2\right] \frac{L(L-1)(L-2)}{2} \alpha e^{-\alpha\gamma} \times \left\{ \frac{\alpha^2\gamma^2}{6} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[ k\alpha\gamma - 3\left(1 - e^{-\frac{\alpha\gamma k}{3}}\right)\right]\right\} d\gamma. \tag{42}$$

The bit error probability becomes [3]

$$\begin{aligned} P_b = \frac{L(L-1)(L-2)}{64} \Bigg\{ & \frac{16}{3\left(\frac{\overline{E}_b}{LN_0}+1\right)^3} + \frac{6\frac{\overline{E}_b}{LN_0}}{\left(\frac{\overline{E}_b}{LN_0}+1\right)^4} + \frac{2\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^5} \\ & + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \Bigg[ \frac{16k}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^2} - \frac{48}{\left(1+\frac{\overline{E}_b}{LN_0}\right)} + \frac{144}{\left(3+k+3\frac{\overline{E}_b}{LN_0}\right)} \\ & + \frac{12k\left(\frac{\overline{E}_b}{LN_0}\right)}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^3} - \frac{18\frac{\overline{E}_b}{LN_0}}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^2} + \frac{162\frac{\overline{E}_b}{LN_0}}{\left(3+k+3\frac{\overline{E}_b}{LN_0}\right)^2} + \frac{3k\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^4} \\ & - \frac{3\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(1+\frac{\overline{E}_b}{LN_0}\right)^3} + \frac{81\left(\frac{\overline{E}_b}{LN_0}\right)^2}{\left(3+k+3\frac{\overline{E}_b}{LN_0}\right)^3} \Bigg]\Bigg\}. \end{aligned} \tag{43}$$

In the next chapter we derive the bit error probability when first order post-detection selection combining is employed.

# III. FIRST ORDER POST-DETECTION SELECTION COMBINING (PDSC-1) ANALYSIS

In the first-order Post-Detection Selection Combining (PDSC-1) technique [14], the maximum of L diversity samples at the output of the L noncoherent DPSK demodulators are considered. The diversity receiver is shown in Fig.3.

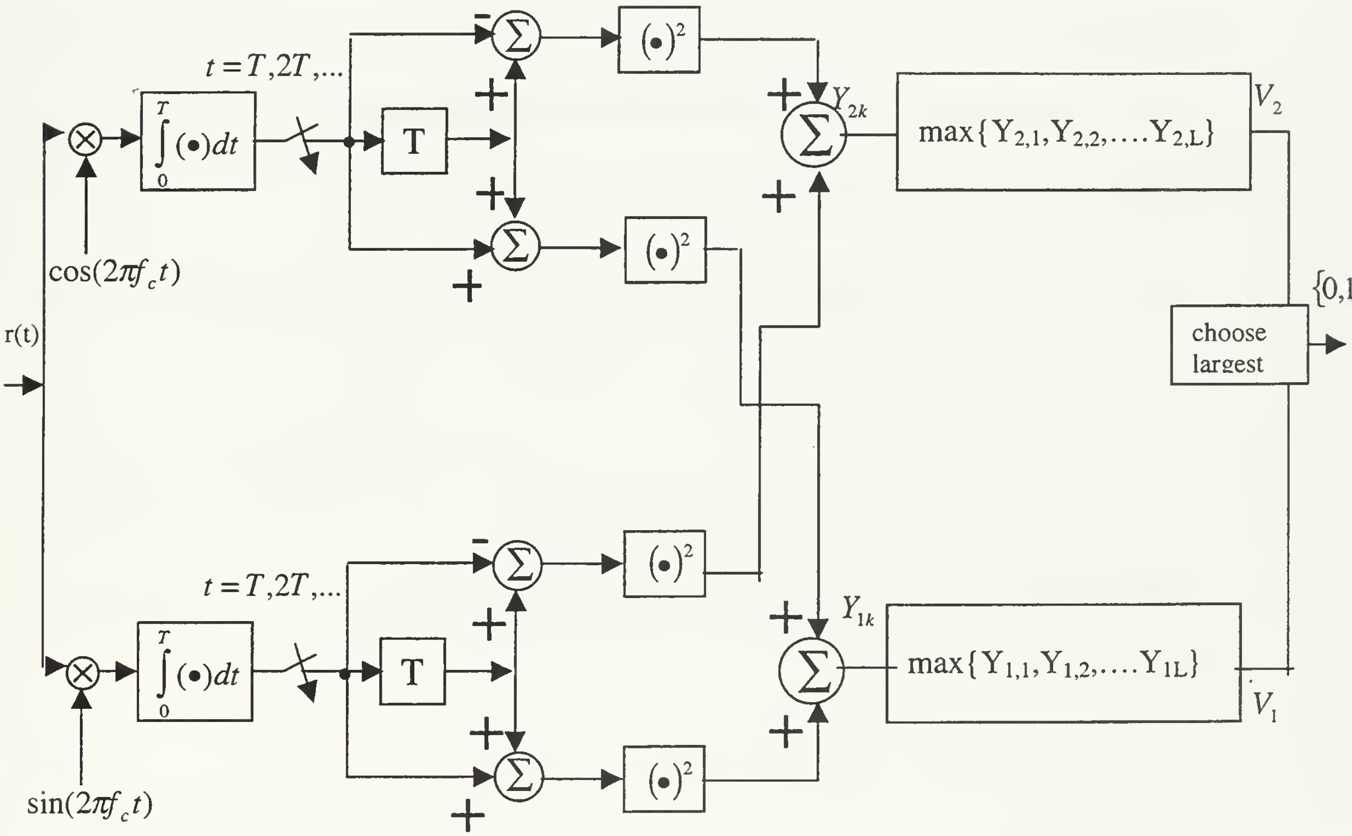


Figure 3. Block diagram of a noncoherent DPSK receiver with first order Post - Detection Selection Combining

## A. PROBABILITY DENSITY FUNCTIONS OF DECISION VARIABLES

As shown in Fig. 3 $V_1$ and $V_2$ denote the decision variables for the signal branch and non-signal branch, respectively. We have

$$V_1 = \max\{Y_{1,1}, Y_{1,2} \ldots Y_{1,L}\}, \tag{44}$$

$$V_2 = \max\{Y_{2,1}, Y_{2,2} \ldots Y_{2,L}\}, \tag{45}$$

where $Y_{1k}$ and $Y_{2k}$, $k = 1,2,\ldots L$ are the amplitudes at the output of the DPSK demodulator of the k-th diversity channel. We assume that $Y_{1k}$ and $Y_{2k}$ are independent and identically distributed random variables for all k.

For the signal branch, the probability density function of the k-th signal amplitude $Y_{1k}$ is a central chi-square density function given in (20)

$$f_{Y_{1k}}(y_{1k}) = \frac{1}{2\sigma_1^2} \exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right), \qquad y_{1k} \geq 0, \tag{46}$$

where $\sigma_1^2$ is given in (21) and (22)

$$\sigma_1^2 = 4\sigma^2 + N_0 = 2\overline{E} + N_0 . \tag{47}$$

The probabilty density function of the decision variable $V_1$ in (44) is given by (Appendix A)

$$f_{V_1}(v_1) = L\, f_{Y_{1k}}(v_1)\left[F_{Y_{1k}}(v_1)\right]^{L-1}, \tag{48}$$

where

$$F_{Y_{1k}}(v_1) = \int_0^{v_1} \frac{1}{2\sigma_1^2} \exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right) dy_{1k} = 1 - \exp\left(-\frac{v_1}{2\sigma_1^2}\right). \tag{49}$$

Substituting (46) and (49) into (48) yields

$$f_{V_1}(v_1)=\frac{L}{2\sigma_1^2}\exp\left(-\frac{v_1}{2\sigma_1^2}\right)\left[1-\exp\left(-\frac{v_1}{2\sigma_1^2}\right)\right]^{L-1}. \tag{50}$$

Similarly, for the non-signal branch, the probability density function of the k-th noise amplitude $Y_{2k}$ is a central chi-square density function given in (25)

$$f_{Y_{2k}}(y_{2k})=\frac{1}{2\sigma_2^2}\exp\left(-\frac{y_{2k}}{2\sigma_2^2}\right), \qquad y_{2k}\geq 0, \tag{51}$$

where $\sigma_2^2=\sigma_n^2=N_0$. The probability density function of $V_2$ in (45) is given by (Appendix A)

$$f_{V_2}(v_2)=L\ f_{Y_{2k}}(v_2)\left[F_{Y_{2k}}(v_2)\right]^{L-1}, \tag{52}$$

where

$$F_{Y_{2k}}(v_2)=\int_0^{v_2}\frac{1}{2\sigma_2^2}\exp\left(-\frac{y_{2k}}{2\sigma_2^2}\right)dy_{2k}=1-\exp\left(-\frac{v_2}{2\sigma_2^2}\right). \tag{53}$$

Substituting (51) and (53) into (52) we have

$$f_{V_2}(v_2)=\frac{L}{2\sigma_2^2}\exp\left(-\frac{v_2}{2\sigma_2^2}\right)\left[1-\exp\left(-\frac{v_2}{2\sigma_2^2}\right)\right]^{L-1}. \tag{54}$$

## B. BIT ERROR PROBABILITY

Using the results derived in Part A, the probability of error for first order Post-Detection Selection Combining is

$$P_e=\int_0^{\infty}\left[\int_{v_1}^{\infty}f_{V_2}(v_2)\,dv_2\right]f_{V_1}(v_1)\,dv_1. \tag{55}$$

Substituting (50) and (54) into (55) yields

Substituting (50) and (54) into (55) yields

$$P_e = \int_0^\infty \left[ \int_{v_1}^\infty \frac{L}{2\sigma_2^2} \exp\left(-\frac{v_2}{2\sigma_2^2}\right)\left(1-\exp\left(-\frac{v_2}{2\sigma_2^2}\right)\right)^{L-1} dv_2 \right]$$
$$\times \frac{L}{2\sigma_1^2} \exp\left(-\frac{v_1}{2\sigma_1^2}\right)\left(1-\exp\left(-\frac{v_1}{2\sigma_1^2}\right)\right)^{L-1} dv_1 . \qquad (56)$$

Applying the binomial theorem,

$$(a+b)^N = \sum_{k=0}^{N} \binom{N}{k} a^{N-k} b^k , \qquad (57)$$

to the exponential terms in (56), the exponential terms can be simplified as

$$\left[1-\exp\left(-\frac{v_2}{2\sigma_2^2}\right)\right]^{L-1} = \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \left(\exp\left(-\frac{v_2}{2\sigma_2^2}\right)\right)^k \qquad (58)$$

and

$$\left[1-\exp\left(-\frac{v_1}{2\sigma_1^2}\right)\right]^{L-1} = \sum_{j=0}^{L-1} \binom{L-1}{j} (-1)^j \left(\exp\left(-\frac{v_1}{2\sigma_1^2}\right)\right)^j . \qquad (59)$$

Then, (56) becomes

$$P_e = \int_0^\infty \left[ \int_{v_1}^\infty \frac{L}{2\sigma_2^2} \sum_{k=0}^{L-1} \binom{L-1}{k} (-1)^k \left(\exp\left(-v_2\left(\frac{1+k}{2\sigma_2^2}\right)\right)\right) dv_2 \right]$$
$$\times \frac{L}{2\sigma_1^2} \sum_{j=0}^{L-1} \binom{L-1}{j} (-1)^j \left(\exp\left(-v_1\left(\frac{1+j}{2\sigma_1^2}\right)\right)\right) dv_1 . \qquad (60)$$

Performing the integration in (60), the bit error probability for PDSC-1 is found to be

$$P_e = \sum_{k=0}^{L-1} \sum_{j=0}^{L-1} \binom{L}{k+1} \frac{(L-1)!\,L(-1)^{j+k}}{(L-1-j)!\,j!\,(1+j)} \cdot \frac{1}{\frac{1+k}{1+j} \cdot \frac{\sigma_1^2}{\sigma_2^2} + 1} . \qquad (61)$$

Rewriting the parameters $\sigma_1$ and $\sigma_2$ as

$$\sigma_1^2 = 4\sigma^2 + N_0 = 2\overline{E} + N_0 \tag{62}$$

and

$$\sigma_2^2 = \sigma_n^2 = N_0 \ , \tag{63}$$

then, the bit error probability is found in terms of the energy per diversity channel as

$$P_e = \sum_{k=0}^{L-1}\sum_{j=0}^{L-1}\binom{L}{k+1}\binom{L}{j+1}(-1)^{j+k}\frac{1}{1+\left(1+\frac{2\overline{E}}{N_0}\right)\cdot\left(\frac{1+k}{1+j}\right)} . \tag{64}$$

Substituting $\overline{E}$ by $\dfrac{\overline{E}_b}{L}$ in (64), the bit error probability for PDSC-1 is found to be

$$P_b = \sum_{k=0}^{L-1}\sum_{j=0}^{L-1}\binom{L}{k+1}\binom{L}{j+1}(-1)^{j+k}\frac{1}{1+\left(1+\frac{2\overline{E}_b}{LN_0}\right)\cdot\left(\frac{1+k}{1+j}\right)} . \tag{65}$$

In the next chapter we will derive the bit error probability when second order Post-Detection Selection Combining is employed.

# IV. SECOND ORDER POST-DETECTION SELECTION COMBINING (PDSC-2) ANALYSIS

In the second-order Post-Detection Selection Combining (PDSC-2) technique, the two largest values of L diversity samples at the outputs of the L noncoherent DPSK demodulators are considered. The diversity receiver is shown in Fig.4.

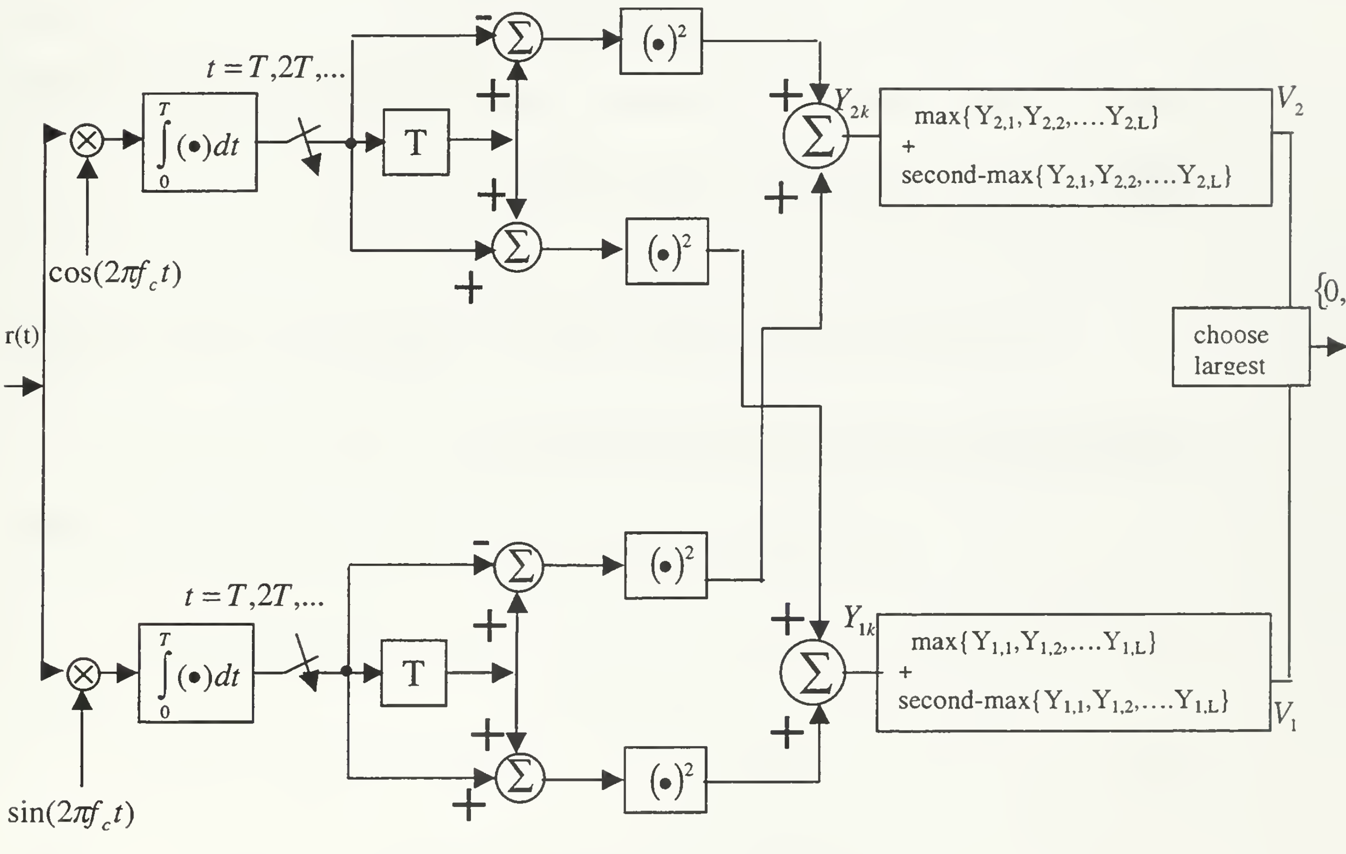


Figure 4. Block diagram of a noncoherent DPSK receiver with second order Post - Detection Selection Combining

## A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES

As shown in Fig. 4 let $V_1$ and $V_2$ denote the decision variables for the signal branch and non-signal branch, respectively. For the signal branch, let $V_{1,1}$ and $V_{1,2}$ denote the two dependent random variables that represent the two largest values at the output of the L noncoherent DPSK demodulators, respectively

$$V_{1,1} = \max\{Y_{1,1}, Y_{1,2} \ldots Y_{1,L}\}, \tag{66}$$

$$V_{1,2} = \text{second}\max\{Y_{1,1}, Y_{1,2} \ldots Y_{1,L}\}. \tag{67}$$

Also, let $V_{2,1}$ and $V_{2,2}$ denote the two dependent random variables that represent the two largest values at the output of the non-signal branch

$$V_{2,1} = \max\{Y_{2,1}, Y_{2,2} \ldots Y_{2,L}\}, \tag{68}$$

$$V_{2,2} = \text{second}\max\{Y_{2,1}, Y_{2,2} \ldots Y_{2,L}\}. \tag{69}$$

Then, the decision variables $V_1$ and $V_2$ can be written as

$$V_1 = \max\{Y_{1,1}, Y_{1,2} \ldots Y_{1,L}\} + \text{second max}\{Y_{1,1}, Y_{1,2} \ldots Y_{1,L}\}, \tag{70}$$

or

$$V_1 = V_{1,1} + V_{1,2}, \tag{71}$$

and

$$V_2 = \max\{V_{2,1}, Y_{2,2} \ldots Y_{2,L}\} + \text{second max}\{Y_{2,1}, Y_{2,2} \ldots Y_{2,L}\}, \tag{72}$$

or

$$V_2 = V_{2,1} + V_{2,2}, \tag{73}$$

where $Y_{1k}$ and $Y_{2k}$, $k = 1,2,...L$ are the amplitudes at the outputs of the DPSK demodulator in diversity channel $k$. We assume that $Y_{1k}$ and $Y_{2k}$ are independent and identically distributed random variables.

For the signal branch, the probability density function of the $k\,th$ signal amplitude $Y_{1k}$ is a central chi-square density function given in (20)

$$f_{Y_{1k}}(y_{1k}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right), \qquad y_{1k} \geq 0, \tag{74}$$

where $\sigma_1^2$ is given in (21) and (22) as

$$\sigma_1^2 = 4\sigma^2 + N_0 = 2\overline{E} + N_0. \tag{75}$$

$V_1$ is the sum of $V_{1,1}$ and $V_{1,2}$. In order to calculate the pdf of decision variable $V_1$, we first need to obtain the joint pdf of $V_{1,1}$ and $V_{1,2}$ which is given by (Appendix B)

$$f_{V_{1,1},V_{1,2}}(v_{1,1}, v_{1,2}) = L(L-1)\, f_{Y_{1k}}(v_{1,1})\, f_{Y_{1k}}(v_{1,2}) \left[F_{Y_{1k}}(v_{1,2})\right]^{L-2}, \qquad v_{1,1} \geq v_{1,2}, \tag{76}$$

where

$$f_{Y_{1k}}(v_{1,1}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{v_{1,1}}{2\sigma_1^2}\right), \qquad v_{1,1} \geq 0, \tag{77}$$

$$f_{Y_{1k}}(v_{1,2}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{v_{1,2}}{2\sigma_1^2}\right), \qquad v_{1,2} \geq 0, \tag{78}$$

and

$$F_{Y_{1k}}(v_{1,2}) = \int_0^{v_{1,2}} \frac{1}{2\sigma_1^2}\exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right) dy_{1k} = 1 - \exp\left(-\frac{v_{1,2}}{2\sigma_1^2}\right). \tag{79}$$

The cumulative distribution function (cdf) of $V_1$ is found by integrating the joint density $f_{V_{1,1},V_{1,2}}\left(v_{1,1}, v_{1,2}\right)$ over the region shown in [3]. We obtain

$$\begin{aligned} F_{V_1}(v_1) &= \int_0^{v_1/2} \int_{v_{1,2}}^{v_1 - v_{1,2}} f_{V_{1,1},V_{1,2}}\left(v_{1,1}, v_{1,2}\right) dv_{1,1}\, dv_{1,2} \qquad \qquad for\ v_{1,1} \geq v_{1,2} \\ &= \int_0^{v_1/2} \int_{v_{1,2}}^{v_1 - v_{1,2}} \frac{L(L-1)}{4\sigma_1^4} \exp\left(-\frac{v_{1,1}+v_{1,2}}{2\sigma_1^2}\right)\left[1-\exp\left(-\frac{v_{1,2}}{2\sigma_1^2}\right)\right]^{L-2} dv_{1,1}\, dv_{1,2}. \end{aligned} \tag{80}$$

Performing this integration yields [3]

$$F_{V_1}(v_1) = L(L-1)\left\{\frac{1}{2}\left[1-\left(1+\frac{v_1}{2\sigma_1^2}\right)e^{-v_1/2\sigma_1^2}\right] + \sum_{j=1}^{L-2}\binom{L-2}{j}(-1)^j W_1(j)\right\}, \tag{81}$$

where

$$W_1(j) = \frac{1}{2+j} - \frac{1}{j}e^{-v_1/2\sigma_1^2} + \frac{2}{j(2+j)}\exp\left(-\frac{v_1(2+j)}{4\sigma_1^2}\right). \tag{82}$$

Differentiating (81) with respect to $v_1$, $f_{V_1}(v_1) = \dfrac{d\,F_{V_1}(v_1)}{dv_1}$, yields the pdf of the decision variable for the signal branch as follows [3]

$$f_{V_1}(v_1) = L(L-1)\frac{\exp\left(-\dfrac{v_1}{2\sigma_1^2}\right)}{2\sigma_1^2}\left[\frac{v_1}{4\sigma_1^2} + \sum_{j=1}^{L-2}\binom{L-2}{j}\frac{(-1)^j}{j}\left(1-\exp\left(-\frac{v_1 j}{4\sigma_1^2}\right)\right)\right]. \tag{83}$$

Similarly, for the non-signal branch, the probability density function of the k-th noise amplitude $Y_{2k}$ is a central chi-square density function given in (25)

$$f_{Y_{2k}}(y_{2k}) = \frac{1}{2\sigma_2^2}\exp\left(-\frac{y_{2k}}{2\sigma_2^2}\right), \qquad \qquad y_{2k} \geq 0, \tag{84}$$

where $\sigma_2^2 = \sigma_n^2 = N_0$. The joint pdf of $V_{2,1}$ and $V_{2,2}$ is given by (Appendix B)

$$f_{V_{2,1},V_{2,2}}(v_{2,1}, v_{2,2}) = L(L-1)\, f_{Y_{2k}}(v_{2,1})\, f_{Y_{2k}}(v_{2,2}) \left[F_{Y_{2k}}(v_{2,2})\right]^{L-2}, \qquad v_{2,1} \geq v_{2,2}, \tag{85}$$

where

$$f_{Y_{2k}}(v_{2,1}) = \frac{1}{2\sigma_2^2} \exp\left(-\frac{v_{2,1}}{2\sigma_2^2}\right), \qquad v_{2,1} \geq 0, \tag{86}$$

$$f_{Y_{2k}}(v_{2,2}) = \frac{1}{2\sigma_2^2} \exp\left(-\frac{v_{2,2}}{2\sigma_2^2}\right), \qquad v_{2,2} \geq 0, \tag{87}$$

and

$$F_{Y_{2k}}(v_{2,2}) = \int_0^{v_{2,2}} \frac{1}{2\sigma_2^2} \exp\left(-\frac{y_{2k}}{2\sigma_2^2}\right) dy_{2k} = 1 - \exp\left(-\frac{v_{2,2}}{2\sigma_2^2}\right). \tag{88}$$

To find the pdf of $V_2 = V_{2,1} + V_{2,2}$, we must first obtain the cdf of $V_2$. By integrating the joint density $f_{V_{2,1},V_{2,2}}(v_{2,1}, v_{2,2})$ over the region shown in [3] we obtain

$$\begin{aligned} F_{V_2}(v_2) &= \int_0^{v_2/2} \int_{v_{2,2}}^{v_2 - v_{2,2}} f_{V_{2,1},V_{2,2}}(v_{2,1}, v_{2,2})\, dv_{2,1}\, dv_{2,2} \qquad \text{for } v_{2,1} > v_{2,2} \\ &= \int_0^{v_2/2} \int_{v_{2,2}}^{v_2 - v_{2,2}} \frac{L(L-1)}{4\sigma_2^4} \exp\left(-\frac{v_{2,1}+v_{2,2}}{2\sigma_2^2}\right) \left[1 - \exp\left(-\frac{v_{2,2}}{2\sigma_2^2}\right)\right]^{L-2} dv_{2,1}\, dv_{2,2}. \end{aligned} \tag{89}$$

Performing this integration yields [3]

$$F_{V_2}(v_2) = L(L-1) \left\{ \frac{1}{2}\left[1 - \left(1 + \frac{v_2}{2\sigma_2^2}\right) e^{-v_2/2\sigma_2^2}\right] + \sum_{k=1}^{L-2} \binom{L-2}{k} (-1)^k\, W_2(k) \right\}, \tag{90}$$

where

$$W_2(k)=\frac{1}{2+k}-\frac{1}{k}e^{-v_2/2\sigma_2^2}+\frac{2}{k(2+k)}\exp\left(-\frac{v_2(2+k)}{4\sigma_2^2}\right). \quad (91)$$

Differentiating (90) with respect to $v_2$, $f_{V_2}(v_2)=\dfrac{d\,F_{V_2}(v_2)}{dv_2}$, yields the pdf of the decision variable for the non-signal branch as follows [3]

$$f_{V_2}(v_2)=L(L-1)\frac{\exp\left(-\dfrac{v_2}{2\sigma_2^2}\right)}{2\sigma_2^2}\left[\frac{v_2}{4\sigma_2^2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left(1-\exp\left(-\frac{v_2\,k}{4\sigma_2^2}\right)\right)\right]. \quad (92)$$

## B. BIT ERROR PROBABILITY

Using the results derived in Part A, the probability of error for second-order Post-Detection Selection Combining is

$$P_e=\int_0^\infty\left[\int_{v_1}^\infty f_{V_2}(v_2)\,dv_2\right]f_{V_1}(v_1)\,dv_1\,. \quad (93)$$

Substituting (83) and (92) into (93) yields

$$\begin{aligned}P_e=\int_0^\infty\int_{v_1}^\infty L^2(L-1)^2\,&\frac{\exp\left(-\dfrac{v_1}{2\sigma_1^2}\right)}{2\sigma_1^2}\left[\frac{v_1}{4\sigma_1^2}+\sum_{j=1}^{L-2}\binom{L-2}{j}\frac{(-1)^j}{j}\left(1-\exp\left(-\frac{v_1\,j}{4\sigma_1^2}\right)\right)\right]\\ &\times\frac{\exp\left(-\dfrac{v_2}{2\sigma_2^2}\right)}{2\sigma_2^2}\left[\frac{v_2}{4\sigma_2^2}+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left(1-\exp\left(-\frac{v_2\,k}{4\sigma_2^2}\right)\right)\right]dv_2\,dv_1\,.\end{aligned} \quad (94)$$

The result is found in terms of $\sigma_1^2$ and $\sigma_2^2$ as

$$P_e = L^2(L-1)^2\left\{\frac{\sigma_1^2\sigma_2^4}{2(\sigma_1^2+\sigma_2^2)^3}+\frac{\sigma_2^4}{4(\sigma_1^2+\sigma_2^2)^2}\right.$$

$$+\sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[\frac{\sigma_2^4}{2(\sigma_1^2+\sigma_2^2)^2}-\frac{4\sigma_2^4}{(2+k)(\sigma_1^2(2+k)+2\sigma_2^2)^2}\right]$$

$$+\sum_{j=1}^{L-2}\binom{L-2}{j}\frac{(-1)^j}{j}\left[\begin{array}{l}\dfrac{\sigma_1^2\sigma_2^2}{2(\sigma_1^2+\sigma_2^2)^2}-\dfrac{2\sigma_1^2\sigma_2^2}{(2\sigma_1^2+\sigma_2^2(2+j))^2}\\ +\dfrac{\sigma_2^2}{2(\sigma_1^2+\sigma_2^2)}-\dfrac{\sigma_2^2}{2\sigma_1^2+\sigma_2^2(2+j)}\end{array}\right]$$

$$+\sum_{k=1}^{L-2}\sum_{j=1}^{L-2}\binom{L-2}{k}\binom{L-2}{j}\frac{(-1)^{j+k}}{jk}$$

$$\left.\times\left[\begin{array}{l}\dfrac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}-\dfrac{2\sigma_2^2}{2\sigma_1^2+\sigma_2^2(2+j)}-\\ \dfrac{4\sigma_2^2}{(2+k)(2\sigma_2^2+\sigma_1^2(2+k))}+\dfrac{4\sigma_2^2}{(2+k)(\sigma_1^2(2+k)+\sigma_2^2(2+j))}\end{array}\right]\right\}. \tag{95}$$

By rewriting the parameters $\sigma_1$ and $\sigma_2$ as

$$\sigma_1^2 = 4\sigma^2 + N_0 = 2\overline{E} + N_0 \tag{96}$$

and

$$\sigma_2^2 = \sigma_n^2 = N_0, \tag{97}$$

the bit error probability is found in terms of the energy per diversity channel as

$$P_e = L^2(L-1)^2 \left\{ \frac{\frac{2\overline{E}}{N_0}+1}{2\left(\frac{2\overline{E}}{N_0}+2\right)^3} + \frac{1}{4\left(\frac{2\overline{E}}{N_0}+2\right)^2} \right.$$

$$+ \sum_{k=1}^{L-2} \binom{L-2}{k} \frac{(-1)^k}{k} \left[ \frac{1}{2\left(\frac{2\overline{E}}{N_0}+2\right)^2} - \frac{4}{(2+k)\left[\left(\frac{2\overline{E}}{N_0}+1\right)(2+k)+2\right]^2} \right]$$

$$+ \sum_{j=1}^{L-2} \binom{L-2}{j} \frac{(-1)^j}{j} \left[ \begin{array}{l} \frac{\left(\frac{2\overline{E}}{N_0}+1\right)}{2\left(\frac{2\overline{E}}{N_0}+2\right)^2} - \frac{2\left(\frac{2\overline{E}}{N_0}+1\right)}{\left(\frac{4\overline{E}}{N_0}+4+j\right)} \\ + \frac{1}{2\left(\frac{2\overline{E}}{N_0}+2\right)} - \frac{1}{\left(\frac{4\overline{E}}{N_0}+4+j\right)} \end{array} \right]$$

$$+ \sum_{k=1}^{L-2} \sum_{j=1}^{L-2} \binom{L-2}{k} \binom{L-2}{j} \frac{(-1)^{j+k}}{jk}$$

$$\left. \times \left[ \begin{array}{l} \frac{1}{\frac{2\overline{E}}{N_0}+2} - \frac{2}{\frac{4\overline{E}}{N_0}+4+j} - \frac{4}{(2+k)\left[2+\left(\frac{2\overline{E}}{N_0}+1\right)(2+k)\right]} \\ + \frac{4}{(2+k)\left[\left(\frac{2\overline{E}}{N_0}+1\right)(2+k)+(2+j)\right]} \end{array} \right] \right\}. \tag{98}$$

Substituting $\overline{E}$ by $\frac{\overline{E_b}}{L}$ in (98), the bit error probability becomes

$$
\begin{aligned}
P_b = L^2(L-1)^2 \Bigg\{ & \frac{\frac{2\bar{E}_b}{LN_0}+1}{2\left(\frac{2\bar{E}_b}{LN_0}+2\right)^3} + \frac{1}{4\left(\frac{2\bar{E}_b}{LN_0}+2\right)^2} \\
& + \sum_{k=1}^{L-2}\binom{L-2}{k}\frac{(-1)^k}{k}\left[\frac{1}{2\left(\frac{2\bar{E}_b}{LN_0}+2\right)^2} - \frac{4}{(2+k)\left[\left(\frac{2\bar{E}_b}{LN_0}+1\right)(2+k)+2\right]^2}\right] \\
& + \sum_{j=1}^{L-2}\binom{L-2}{j}\frac{(-1)^j}{j}\left[\begin{array}{l}\dfrac{\left(\frac{2\bar{E}_b}{LN_0}+1\right)}{2\left(\frac{2\bar{E}_b}{LN_0}+2\right)^2} - \dfrac{2\left(\frac{2\bar{E}_b}{LN_0}+1\right)}{\left(\frac{4\bar{E}_b}{LN_0}+4+j\right)} \\ + \dfrac{1}{2\left(\frac{2\bar{E}_b}{LN_0}+2\right)} - \dfrac{1}{\left(\frac{4\bar{E}_b}{LN_0}+4+j\right)}\end{array}\right] \\
& + \sum_{k=1}^{L-2}\sum_{j=1}^{L-2}\binom{L-2}{k}\binom{L-2}{j}\frac{(-1)^{j+k}}{jk} \\
& \times \left[\begin{array}{l}\dfrac{1}{\frac{2\bar{E}_b}{LN_0}+2} - \dfrac{2}{\frac{4\bar{E}_b}{LN_0}+4+j} - \dfrac{4}{(2+k)\left[2+\left(\frac{2\bar{E}_b}{LN_0}+1\right)(2+k)\right]} \\ + \dfrac{4}{(2+k)\left[\left(\frac{2\bar{E}_b}{LN_0}+1\right)(2+k)+(2+j)\right]}\end{array}\right]\Bigg\}. \qquad (99)
\end{aligned}
$$

In the next chapter we will derive the bit error probability when third order Post-Detection Selection Combining is employed.

# V. THIRD ORDER POST-DETECTION SELECTION COMBINING (PDSC-3) ANALYSIS

In the third-order Post-Detection Selection Combining (PDSC-3) technique, the three largest values of L diversity samples at the outputs of the L noncoherent DPSK demodulators are considered. The diversity receiver is shown in Fig.5.

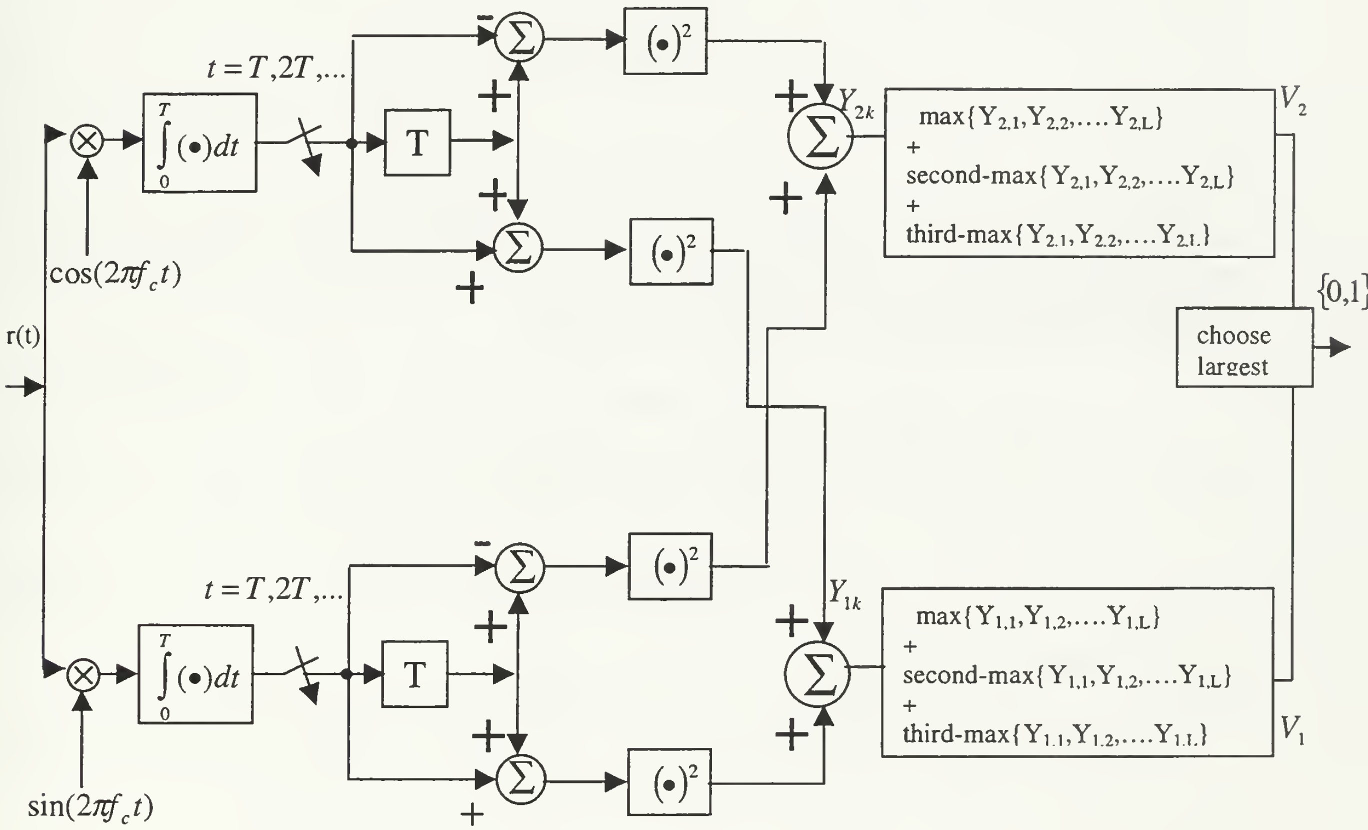


Figure 5. Block diagram of a noncoherent DPSK receiver with third order Post - Detection Selection Combining

## A. PROBABILITY DENSITY FUNCTIONS OF THE DECISION VARIABLES

As shown in Fig. 5 let $V_1$ and $V_2$ denote the decision variables for the signal branch and non-signal branch, respectively,. For the signal branch, let $V_{1,1}, V_{1,2}$ and $V_{1,3}$ denote the three dependent random variables that represent the three largest values at the output of the L noncoherent DPSK demodulators, respectively

$$V_{1,1} = \max\{Y_{1,1}, Y_{1,2} \dots Y_{1,L}\}, \tag{100}$$

$$V_{1,2} = \text{second } \max\{Y_{1,1}, Y_{1,2} \dots Y_{1,L}\}, \tag{101}$$

$$V_{1,3} = \text{third } \max\{Y_{1,1}, Y_{1,2} \dots Y_{1,L}\}. \tag{102}$$

Also, let $V_{2,1}, V_{2,1}$ and $V_{2,3}$ denote the three dependent random variables that represent the three largest values at the output of the non-signal branch

$$V_{2,1} = \max\{Y_{2,1}, Y_{2,2} \dots Y_{2,L}\}, \tag{103}$$

$$V_{2,2} = \text{second } \max\{Y_{2,1}, Y_{2,2} \dots Y_{2,L}\}, \tag{104}$$

$$V_{2,3} = \text{third } \max\{Y_{2,1}, Y_{2,2} \dots Y_{2,L}\}. \tag{105}$$

Then, the decision variables $V_1$ and $V_2$ can be written as

$$V_1 = \max\{Y_{1,1}, Y_{1,2} \dots Y_{1,L}\} + \text{second } \max\{Y_{1,1}, Y_{1,2} \dots Y_{1,L}\} + \text{third } \max\{Y_{1,1}, Y_{1,2} \dots Y_{1,L}\}, \tag{106}$$

or

$$V_1 = V_{1,1} + V_{1,2} + V_{1,3}, \tag{107}$$

and

$$V_2 = \max\{V_{2,1}, Y_{2,2} \dots Y_{2,L}\} + \text{second } \max\{Y_{2,1}, Y_{2,2} \dots Y_{2,L}\} + \text{third } \max\{Y_{2,1}, Y_{2,2} \dots Y_{2,L}\}, \tag{108}$$

or

$$V_2 = V_{2,1} + V_{2,2} + V_{2,3}, \tag{109}$$

where $Y_{1k}$ and $Y_{2k}$, $k = 1,2,...L$ are the amplitudes at the outputs of the DPSK demodulator in diversity channel $k$. We assume that $Y_{1k}$'s and $Y_{2k}$'s are independent and identically distributed random variables.

For the signal branch, the probability density function of the k-th signal amplitude $Y_{1k}$ is a central chi-square density function given in (20)

$$f_{Y_{1k}}(y_{1k}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right), \qquad y_{1k} \geq 0, \tag{110}$$

where $\sigma_1^2$ is given in (21) and (22) as

$$\sigma_1^2 = 4\sigma^2 + N_0 = 2\overline{E} + N_0. \tag{111}$$

Since $V_1 = V_{1,1} + V_{1,2} + V_{1,3}$, we need to obtain the joint pdf of $V_{1,1}, V_{1,2}$ and $V_{1,3}$ which is given by (Appendix C)

$$f_{V_{1,1},V_{1,2},V_{1,3}}(v_{1,1}, v_{1,2}, v_{1,3}) = L(L-1)(L-2)\, f_{Y_{1k}}(v_{1,1})\, f_{Y_{1k}}(v_{1,2}) f_{Y_{1k}}(v_{1,3}) \left[F_{Y_{1k}}(v_{1,3})\right]^{L-3}, \qquad v_{1,1} \geq v_{1,2} \geq v_{1,3}, \tag{112}$$

where

$$f_{Y_{1k}}(v_{1,1}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{v_{1,1}}{2\sigma_1^2}\right), \qquad v_{1,1} \geq 0, \tag{113}$$

$$f_{Y_{1k}}(v_{1,2}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{v_{1,2}}{2\sigma_1^2}\right), \qquad v_{1,2} \geq 0, \tag{114}$$

$$f_{Y_{1k}}(v_{1,3}) = \frac{1}{2\sigma_1^2}\exp\left(-\frac{v_{1,3}}{2\sigma_1^2}\right), \qquad v_{1,3} \geq 0, \tag{115}$$

and

$$F_{Y_{1k}}(v_{1,3}) = \int_0^{v_{1,3}} \frac{1}{2\sigma_1^2}\exp\left(-\frac{y_{1k}}{2\sigma_1^2}\right) dy_{1k} = 1 - \exp\left(-\frac{v_{1,3}}{2\sigma_1^2}\right). \tag{116}$$

To find the pdf of $V_1$, we must first obtain the cumulative distribution function (cdf) of $V_1$. By integrating the joint density $f_{V_{1,1},V_{1,2},V_{1,3}}(v_{1,1}, v_{1,2}, v_{1,3})$ over the region where $v_{1,1} \geq v_{1,2} \geq v_{1,3}$ and $v_{1,1} + v_{1,2} + v_{1,3} \leq v_1$ as shown in [3] we obtain

$$F_{V_1}(v_1) = \int_0^{v_1/3} \int_{v_{1,3}}^{(v_1 - v_{1,3})/2} \int_{v_{1,2}}^{v_1 - v_{1,2} - v_{1,3}} f_{V_{1,1},V_{1,2},V_{1,3}}(v_{1,1}, v_{1,2}, v_{1,3})\, dv_{1,1} dv_{1,2} dv_{1,3} \ . \tag{117}$$

Performing the integration and differentiating the result with respect to $v_1$, $f_{V_1}(v_1) = \dfrac{d\,F_{V_1}(v_1)}{dv_1}$, the pdf of the decision variable for the signal branch is given by [3]

$$\begin{aligned} f_{V_1}(v_1) = & \frac{L(L-1)(L-2)}{2} \frac{e^{-\frac{v_1}{2\sigma_1^2}}}{2\sigma_1^2} \\ & \times \left\{ \frac{v_1^2}{6(2\sigma_1^2)^2} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[ \frac{kv_1}{2\sigma_1^2} - 3\left(1 - \exp\left(-\frac{kv_1}{6\sigma_1^2}\right)\right)\right]\right\}. \end{aligned} \tag{118}$$

Similarly, for the non-signal branch, the probability density function of the k-th noise amplitude $Y_{2k}$ is a central chi-square density function given in (25)

$$f_{Y_{2k}}(y_{2k}) = \frac{1}{2\sigma_2^2} \exp\left(-\frac{y_{2k}}{2\sigma_2^2}\right), \qquad y_{2k} \geq 0, \tag{119}$$

where $\sigma_2^2 = \sigma_n^2 = N_0$. The joint pdf of $V_{2,1}, V_{1,2}$ and $V_{2,3}$ is given by (Appendix C)

$$f_{V_{2,1},V_{2,2},V_{2,3}}(v_{2,1}, v_{2,2}, v_{2,3}) = L(L-1)(L-2)\, f_{Y_{2k}}(v_{2,1}) f_{Y_{2k}}(v_{2,2}) f_{Y_{2k}}(v_{2,3}) \left[F_{Y_{2k}}(v_{2,3})\right]^{L-3}, \quad v_{2,1} \geq v_{2,2} \geq v_{2,3}, \tag{120}$$

where

$$f_{Y_{2k}}(v_{2,1}) = \frac{1}{2\sigma_2^2}\exp\left(-\frac{v_{2,1}}{2\sigma_2^2}\right), \qquad v_{2,1} \geq 0, \tag{121}$$

$$f_{Y_{2k}}(v_{2,2}) = \frac{1}{2\sigma_2^2}\exp\left(-\frac{v_{2,2}}{2\sigma_2^2}\right), \qquad v_{2,2} \geq 0, \tag{122}$$

$$f_{Y_{2k}}(v_{2,3}) = \frac{1}{2\sigma_2^2}\exp\left(-\frac{v_{2,3}}{2\sigma_2^2}\right), \qquad v_{2,3} \geq 0, \tag{123}$$

and

$$F_{Y_{2k}}(v_{2,3}) = \int_0^{v_{2,3}} \frac{1}{2\sigma_2^2}\exp\left(-\frac{y_{2k}}{2\sigma_2^2}\right)dy_{2k} = 1-\exp\left(-\frac{v_{2,3}}{2\sigma_2^2}\right) . \tag{124}$$

To find the pdf of $V_2$, we must first obtain the cdf of $V_2$. Integration of the joint density $f_{V_{2,1},V_{2,2},V_{2,3}}(v_{2,1}, v_{2,2}, v_{2,3})$ over the region where $v_{2,1} \geq v_{2,2} \geq v_{2,3}$ and $v_{2,1} + v_{2,2} + v_{2,3} \leq v_2$ [3] leads to

$$F_{V_2}(v_2) = \int_0^{v_2/3} \int_{v_{2,3}}^{(v_2-v_{2,3})/2} \int_{v_{2,2}}^{v_2-v_{2,2}-v_{2,3}} f_{V_{2,1},V_{2,2},V_{2,3}}(v_{2,1}, v_{2,2}, v_{2,3})\, dv_{2,1}dv_{2,2}dv_{2,3} . \tag{125}$$

Performing the integration and differentiating the result with respect to $v_2$, $f_{V_2}(v_2) = \dfrac{d\,F_{V_2}(v_2)}{dv_2}$, the pdf of the decision variable for the non-signal branch is given by [3]

$$\begin{aligned} f_{V_2}(v_2) = {} & \frac{L(L-1)(L-2)}{2} \frac{e^{-\frac{v_2}{2\sigma_2^2}}}{2\sigma_2^2} \\ & \times\left\{\frac{v_2^2}{6(2\sigma_2^2)^2} + \sum_{j=1}^{L-3}\binom{L-3}{j}\frac{(-1)^j}{j^2}\left[\frac{jv_2}{2\sigma_2^2} - 3\left(1-\exp\left(-\frac{jv_2}{6\sigma_2^2}\right)\right)\right]\right\}. \end{aligned} \tag{126}$$

## B. BIT ERROR PROBABILITY

Using the results derived in Part A, the probability of error for third-order Post-Detection Selection Combining is

$$P_e = \int_0^{\infty} \left[ \int_{v_1}^{\infty} f_{V_2}(v_2)\, dv_2 \right] f_{V_1}(v_1)\, dv_1 \ . \tag{127}$$

Substituting (118) and (126) into (127) yields

$$\begin{aligned} P_e = \int_0^{\infty}\int_{v_1}^{\infty} & L^2(L-1)^2(L-2)^2 \frac{1}{16\sigma_1^2\sigma_2^2} \exp\left(-\frac{v_1}{2\sigma_1^2} - \frac{v_2}{2\sigma_2^2}\right) \\ & \times \left\{ \frac{v_1^2}{6(2\sigma_1^2)^2} + \sum_{k=1}^{L-3} \binom{L-3}{k} \frac{(-1)^k}{k^2} \left[ \frac{kv_1}{2\sigma_1^2} - 3\left(1 - e^{-\frac{kv_1}{6\sigma_1^2}}\right)\right]\right\} \\ & \times \left\{ \frac{v_2^2}{6(2\sigma_2^2)^2} + \sum_{j=1}^{L-3} \binom{L-3}{j} \frac{(-1)^j}{j^2} \left[ \frac{jv_2}{2\sigma_2^2} - 3\left(1 - e^{-\frac{jv_2}{6\sigma_2^2}}\right)\right]\right\} dv_2 dv_1 \ . \end{aligned} \tag{128}$$

The probability of error is found in terms of $\sigma_1^2$ and $\sigma_2^2$ as

$$P_e = L^2(L-1)^2(L-2)^2\left[A+B+C+D\right], \tag{129}$$

where

$$A = \frac{\sigma_1^4\sigma_2^6}{6(\sigma_1^2+\sigma_2^2)^5} + \frac{\sigma_1^2\sigma_2^6}{12(\sigma_1^2+\sigma_2^2)^4} + \frac{\sigma_2^6}{36(\sigma_1^2+\sigma_2^2)^3},$$

$$B = \sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\left[\begin{aligned} & \frac{k\sigma_1^4\sigma_2^4}{4(\sigma_1^2+\sigma_2^2)^4} - \frac{\sigma_1^4\sigma_2^2}{4(\sigma_1^2+\sigma_2^2)^3} + \frac{27\sigma_1^4\sigma_2^2}{4(3\sigma_1^2+3\sigma_2^2+k\sigma_2^2)^2} \\ & + \frac{k\sigma_1^2\sigma_2^4}{6(\sigma_1^2+\sigma_2^2)^3} - \frac{\sigma_1^2\sigma_2^2}{4(\sigma_1^2+\sigma_2^2)^2} + \frac{9\sigma_1^2\sigma_2^2}{4(3\sigma_1^2+3\sigma_2^2+k\sigma_2^2)^2} \\ & + \frac{k\sigma_2^4}{12(\sigma_1^2+\sigma_2^2)^2} - \frac{\sigma_2^2}{4(\sigma_1^2+\sigma_2^2)} + \frac{3\sigma_2^2}{4(3\sigma_1^2+3\sigma_2^2+k\sigma_2^2)} \end{aligned}\right],$$

$$C=\sum_{j=1}^{L-3}\binom{L-3}{j}\frac{(-1)^j}{j^2}\left[\begin{array}{l}\frac{j\sigma_1^2\sigma_2^6}{4\left(\sigma_1^2+\sigma_2^2\right)^4}+\frac{j\sigma_2^6}{12\left(\sigma_1^2+\sigma_2^2\right)^3}-\frac{\sigma_2^6}{4\left(\sigma_1^2+\sigma_2^2\right)^3}\\+\frac{81\sigma_2^6}{4(3+j)\left(3\sigma_1^2+3\sigma_2^2+j\sigma_1^2\right)^3}\end{array}\right],$$

and

$$D=\sum_{k=1}^{L-3}\sum_{j=1}^{L-3}\binom{L-3}{k}\binom{L-3}{j}\frac{(-1)^{k+j}}{(jk)^2}\left[\begin{array}{l}\frac{kj\sigma_1^2\sigma_2^4}{2\left(\sigma_1^2+\sigma_2^2\right)^3}-\frac{3j\sigma_1^2\sigma_2^2}{4\left(\sigma_1^2+\sigma_2^2\right)^2}+\frac{27j\sigma_1^2\sigma_2^2}{4\left(3\sigma_1^2+3\sigma_2^2+k\sigma_2^2\right)^2}\\+\frac{kj\sigma_2^4}{4\left(\sigma_1^2+\sigma_2^2\right)^2}-\frac{3j\sigma_2^2}{4\left(\sigma_1^2+\sigma_2^2\right)}+\frac{9j\sigma_2^2}{4\left(3\sigma_1^2+3\sigma_2^2+k\sigma_2^2\right)}\\-\frac{3k\sigma_2^4}{4\left(\sigma_1^2+\sigma_2^2\right)^2}+\frac{9\sigma_2^2}{4\left(\sigma_1^2+\sigma_2^2\right)}-\frac{27\sigma_2^2}{4\left(3\sigma_1^2+3\sigma_2^2+k\sigma_2^2\right)}\\+\frac{81k\sigma_2^4}{4(3+j)\left(3\sigma_1^2+3\sigma_2^2+j\sigma_1^2\right)}-\frac{81\sigma_2^2}{4(3+j)\left(3\sigma_1^2+3\sigma_2^2+j\sigma_1^2\right)}\\+\frac{81\sigma_2^2}{4(3+j)\left(3\sigma_1^2+3\sigma_2^2+j\sigma_1^2+k\sigma_2^2\right)}\end{array}\right].$$

Rewriting the parameters $\sigma_1$ and $\sigma_2$ as

$$\sigma_1^2=4\sigma^2+N_0=2\overline{E}+N_0=2\frac{\overline{E_b}}{L}+N_0\ , \tag{130}$$

and

$$\sigma_2^2=\sigma_n^2=N_0, \tag{131}$$

allows an expression for the bit error probability in terms of $\overline{E_b}/N_0$ as

$$
\begin{aligned}
P_b = {} & \frac{L^2(L-1)^2(L-2)^2}{4}\left\{\frac{1}{144}\frac{3a^2+3ab+2b^2}{b^5}+\sum_{k=1}^{L-3}\binom{L-3}{k}\frac{(-1)^k}{k^2}\right. \\
& \times\left[\frac{1}{48}\frac{3ka(a+1)+2b(2ka-3a^2+k-3)+4b^2(k-3a-6b)}{b^4}+\frac{81}{(3+k)(a(3+k)+3)^3}\right. \\
& \left.+\frac{3\left(9a^2+3a(6b+k)+(6b+k)^2\right)}{(6b+k)^3}\right]+\sum_{k=1}^{L-3}\sum_{j=1}^{L-3}\binom{L-3}{k}\binom{L-3}{j}\frac{(-1)^{k+j}}{(kj)^2} \\
& \times\left[9\frac{3aj+(6b+k)(j-3)}{(6b+k)^3}+\frac{kja+(j-3)kb-3jab+(9-3j)2b^2}{4b^3}\right. \\
& \left.\left.+81\left(\frac{k-((3+j)a+3)}{(3+j)((3+j)a+3)^2}+\frac{1}{(3+j)^2a+(3+j)(3+k)}\right)\right]\right\};
\end{aligned}
\tag{132}
$$

where $a=\left(\dfrac{2\overline{E}_b}{LN_0}+1\right)$ and $b=\left(\dfrac{\overline{E}_b}{LN_0}+1\right)$.

In the next chapter we will present the performance of the various diversity combining techniques previously discussed.

# VI. NUMERICAL RESULTS

All expressions related to the bit error probability and probability density functions of the decision variables are analytically derived. Numerical results are obtained by using MATLAB [13]. A signal-to-noise ratio range of 6-20 dB for is used in the numerical evaluations.

## A. COMBINING TECHNIQUES AND DIVERSITY

We obtained numerical results for Post-Detection Selection Combining, Selection Combining and Equal Gain Combining. As discussed in the preceding chapters, the main subjects of interest are : first, second and third order Post-Detection Selection Combining.

The test cases allow us to compare the combining techniques for different orders of diversity, L. Plots for Post-Detection Selection Combining, Selection Combining and Equal Gain Combining for different orders of diversity are given in Fig.6 through Fig.12. The bit error probability expressions given in [3] are used for the numerical evaluation of Selection Combining.

To see the differences in the performance among these three techniques, we compare PDSC-1, SC-1, and EGC for diversity order L=1, 2, 3, 4 and 5 in Fig.13 through Fig.17. Then we compare PDSC-2, SC-2 and EGC performance for L=2, 3, 4, 5 and 6 in Fig.18 through Fig.22.

Finally, PDSC-3, SC-3 and EGC performance are compared for L=3, 4, 5, 6, 7, 8, 9 and 10 respectively in Fig.23 through Fig.30. In addition to L=3, 4, 5, 6, 7, diversity orders of 8, 9 and 10 are employed to show a slight improvement.

## B. COMPARISONS

We note that EGC performs similarly or better than the other two methods. However, our main objective is to show the performance difference between PDSC and SC .

In Fig.6 through Fig.12, performance curves of PDSC-1, PDSC-2, PDSC-3, SC-1, SC-2, SC-3 and EGC are plotted for different values of diversity orders. It is seen that as the diversity order increases, the performance of the receiver increases.

In Fig.13 through Fig.17, PDSC-1 and SC-1 allow comparison for L=1, 2, 3, 4 and 5. When L=1, (i.e. when there is no diversity), all three methods perform the same as seen in Fig.13. Starting from diversity of L=2, we notice a slight performance difference between PDSC-1 and SC-1, PDSC-1 performs better than SC-1, and worse than EGC as previously stated. As the value of L increases the performance difference between PDSC-1 and SC-1 increases in favor of PDSC-1 as shown in Fig.14, 15, 16 and 17.

The performance of PDSC-2, SC-2 and EGC is illustrated in Fig.18 through Fig.22. Since two signals are combined, these figures are plotted for diversities of 2 through 6.

When L=2, as seen in Fig.18, all combining techniques perform the same as expected. For L=3, there is virtually no difference between the three techniques. In Fig. 20 (i.e., L=4), and in Fig.21 (i.e., L=5), we see that PDSC-2 performs essentially the same as SC-2. Again, EGC performs better than the other two methods. In Fig.22 (i.e.,L=6), PDSC-2 is slightly better than SC-2.

The performance for SC-3, EGC and PDSC-3 is shown in Fig.23 through Fig.30. In order to more clearly see the performance differences, plots are given for diversities of

L=3, 4, 5, 6, 7, 8, 9 and 10. Since three signals are combined for PDSC-3 and SC-3, the performance curves are plotted starting with L=3. For L=3, all three methods perform exactly the same. Although, it is really difficult to distinguish the performance curves in Fig.24-26, very slight differences in the curves indicate that EGC performs better than SC-3, and SC-3 performs slightly better than PDSC-3.

Beginning with Fig.27, evaluated for L=7, we see that PDSC-3 is starting to perform slightly better than SC-3 for an SNR value of 17 dB. As the diversity orders are increased to L=8, 9 and 10 the performance differences between PDSC-3 and SC-3 become more significant, favoring PDSC-3. For L=10, PDSC-3 outperforms SC-3 for SNR values of 12 dB or larger. But the difference is small.

In summary, PDSC-1 performs better than SC-1, while PDSC-2 and PDSC-3 perform essentially the same as SC-2 and SC-3, respectively. EGC, as expected, performs better than the other two techniques. For time diversity, the EGC, PDSC and SC methods require only one receiver. On the other hand, for space and frequency diversities EGC and PDSC methods require L receivers. Although the SC method requires only one receiver for time, space and frequency diversities, it does require a more complex receiver, one which has the pre-detection combiner.

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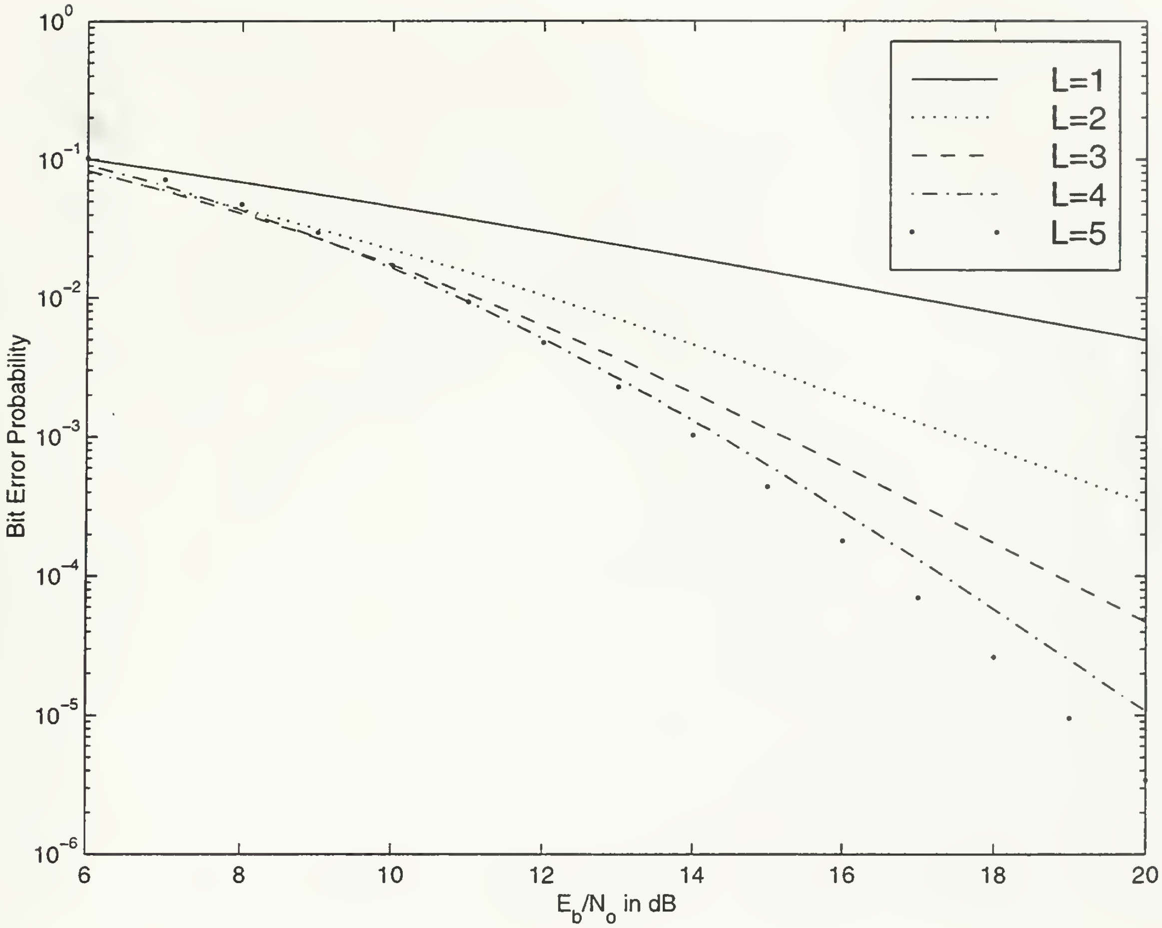


**Figure 6.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using first order post-detection selection combining (PDSC-1) for diversity orders of L=1, 2, 3, 4 and 5.

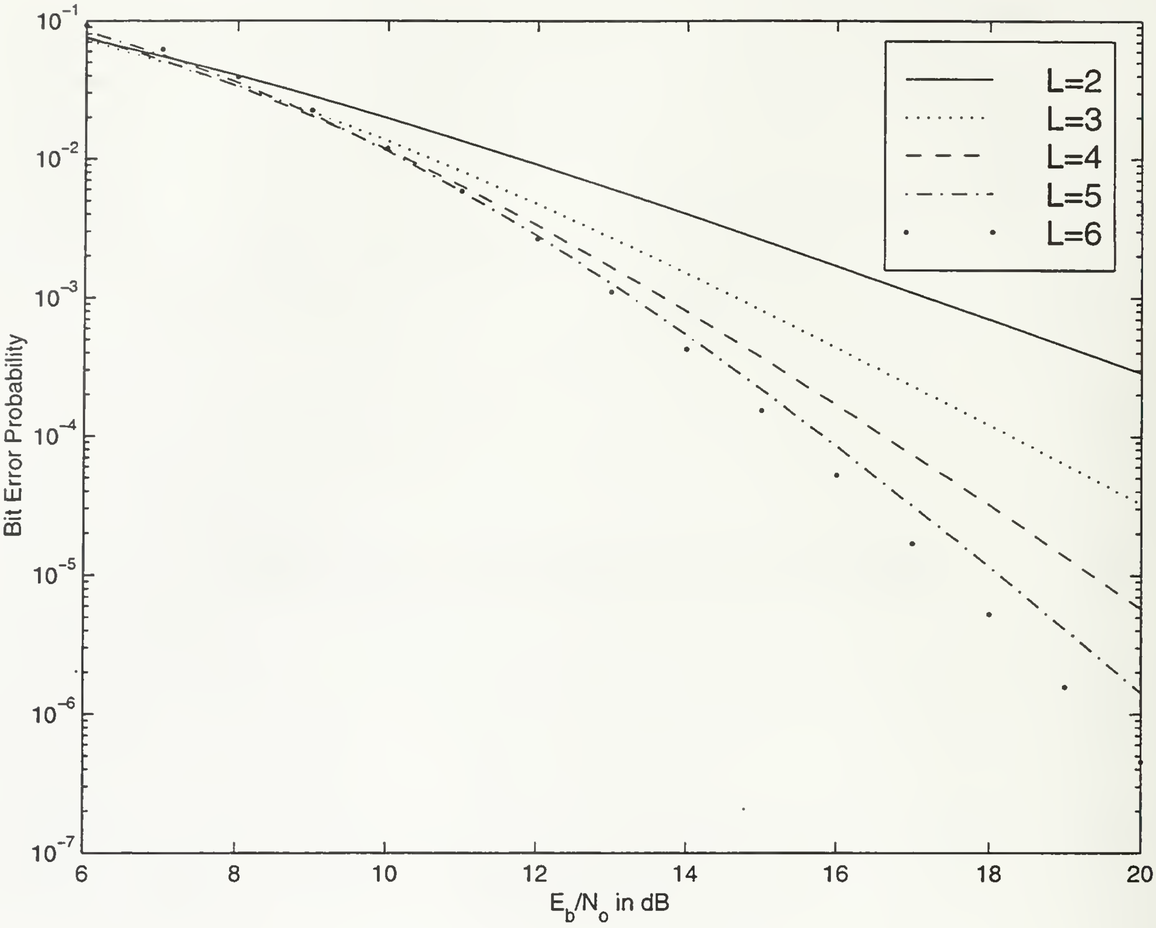


**Figure 7.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using second order post-detection selection combining (PDSC-2) for diversity orders of L=2, 3, 4, 5 and 6.

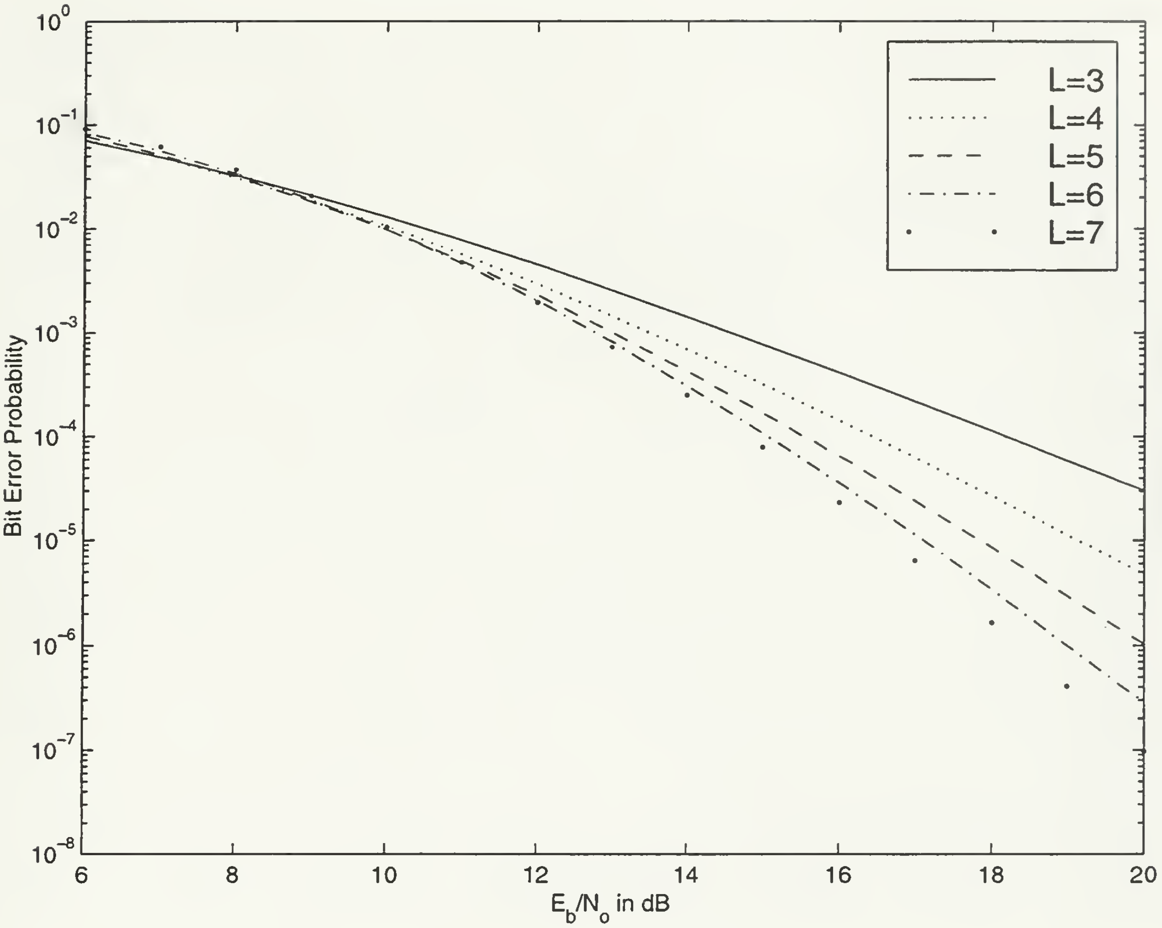


**Figure 8.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using third order post-detection selection combining (PDSC-2) for diversity orders of L=3, 4, 5, 6 and 7.

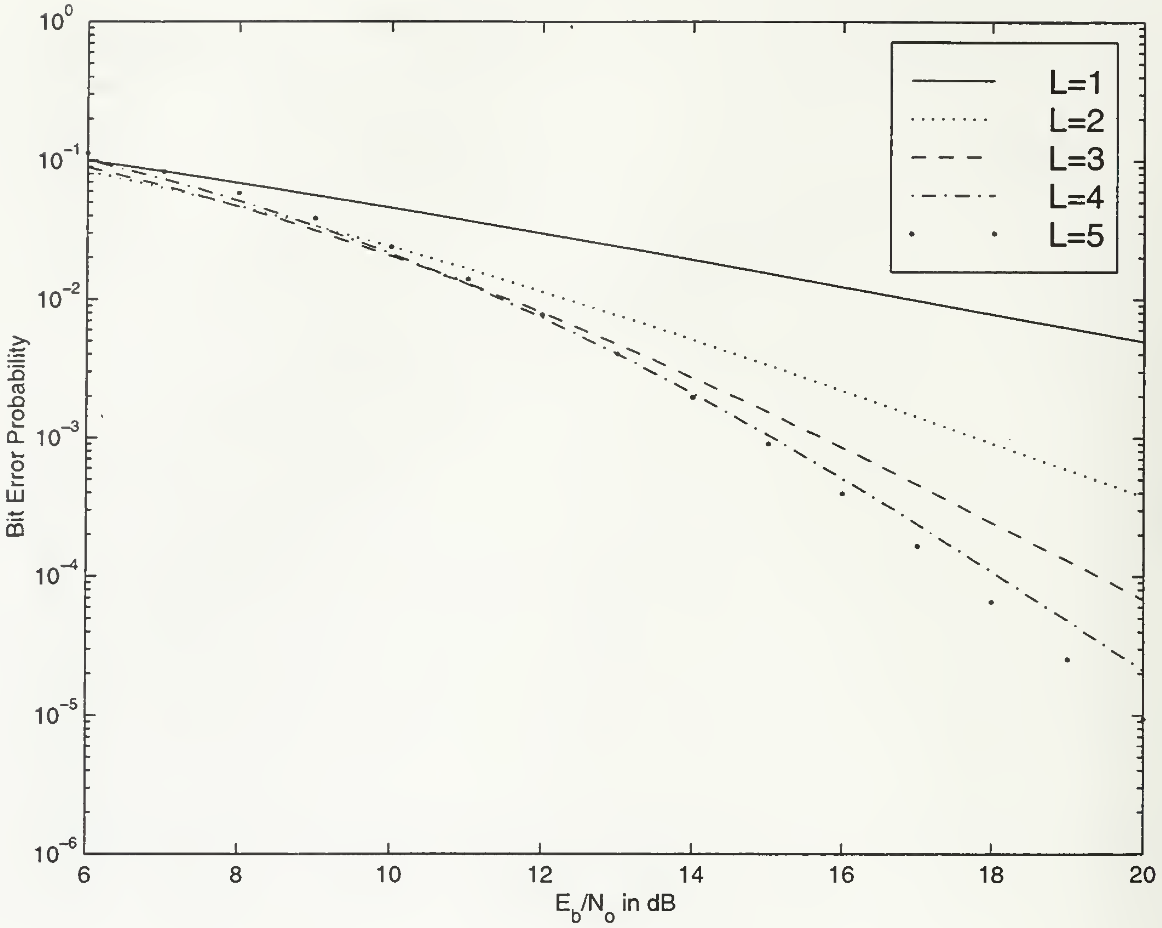


**Figure 9.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using first order selection combining (SC-1) for diversity orders of L=1, 2, 3, 4 and 5.

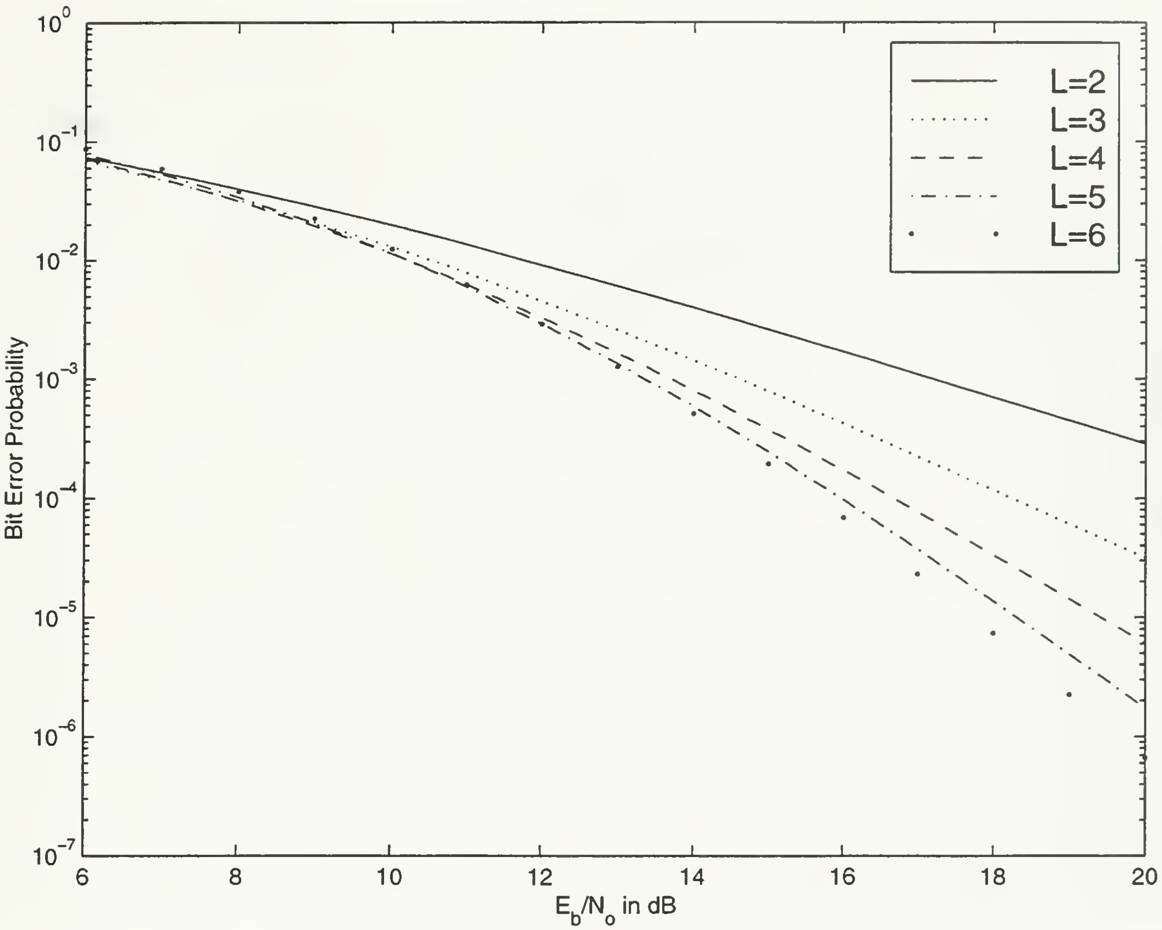


**Figure 10.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using second order selection combining(SC-2) for diversity orders of L=2, 3, 4, 5 and 6.

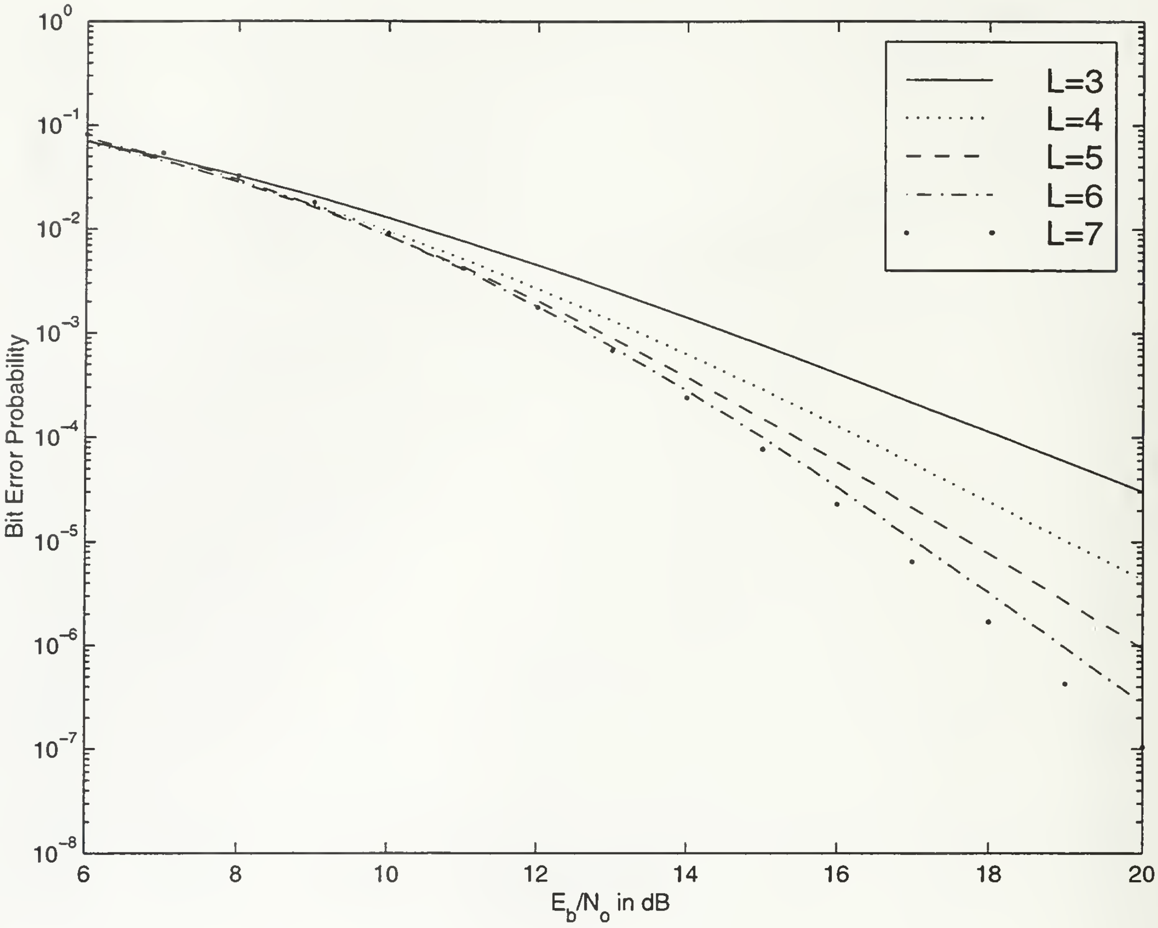


**Figure 11.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using third order selection combining (SC-3) for diversity orders of L=3, 4, 5, 6 and 7.

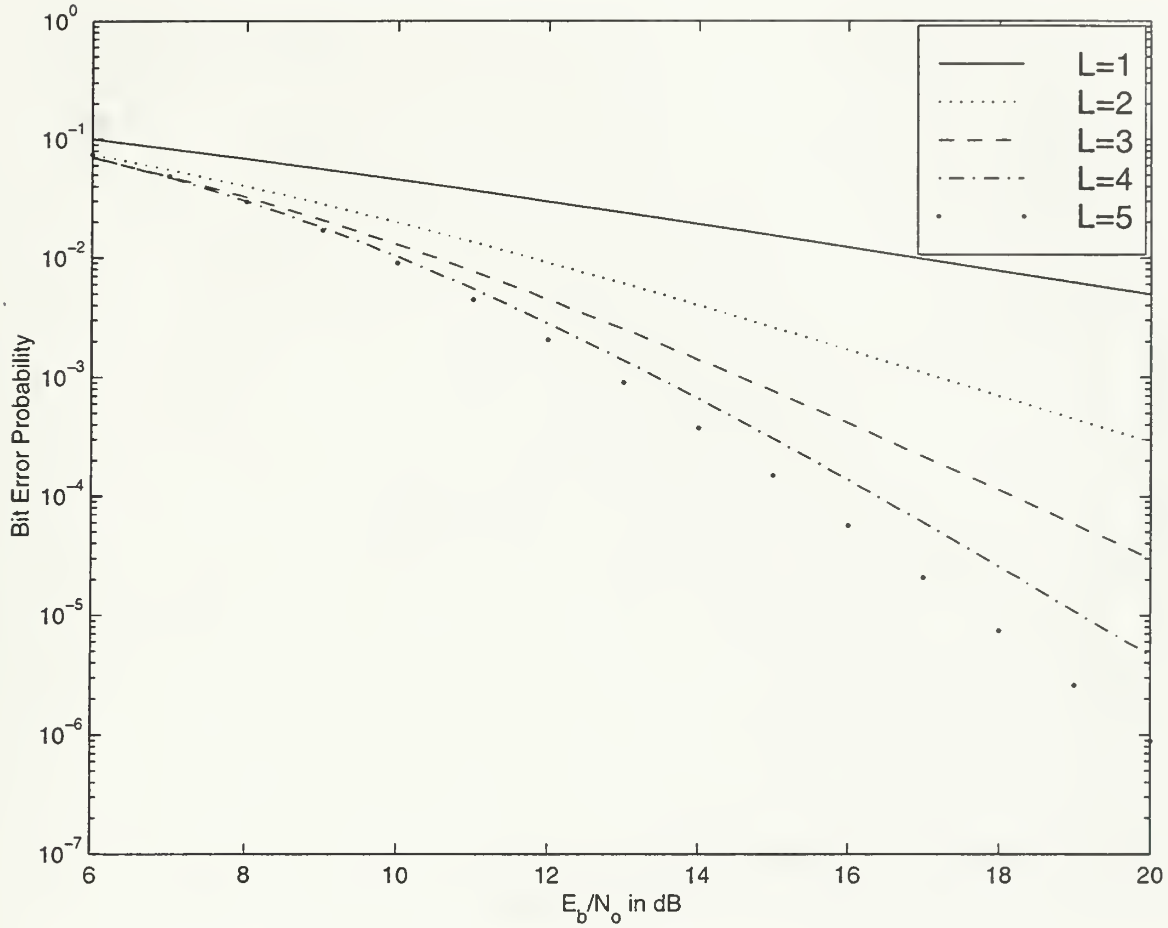


**Figure 12.** Performance of the noncoherent DPSK receiver over a Rayleigh fading channel using equal gain combining (EGC) for diversity orders of L=1, 2, 3, 4 and 5.

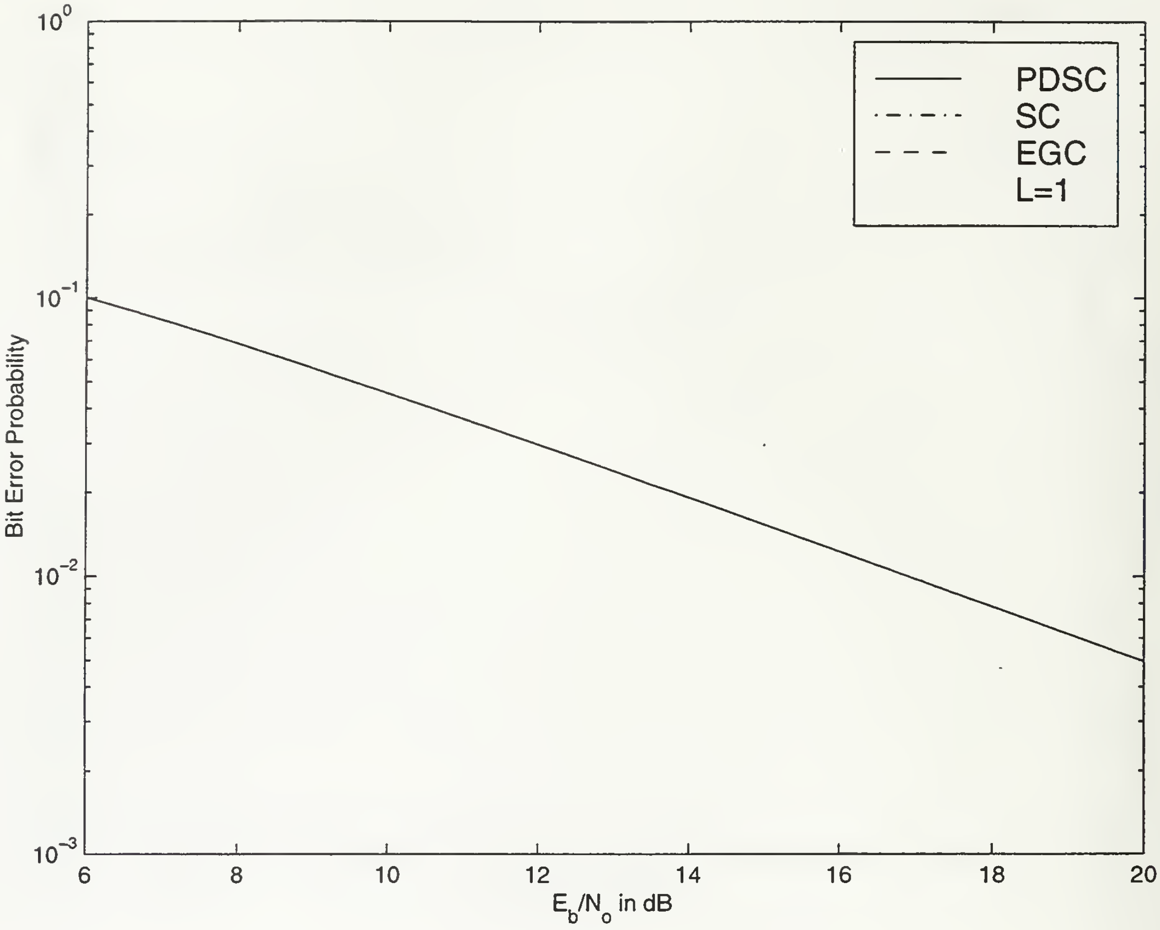


**Figure 13.** Receiver performance of PDSC-1, SC-1, and EGC over a Rayleigh fading channel for L=1.

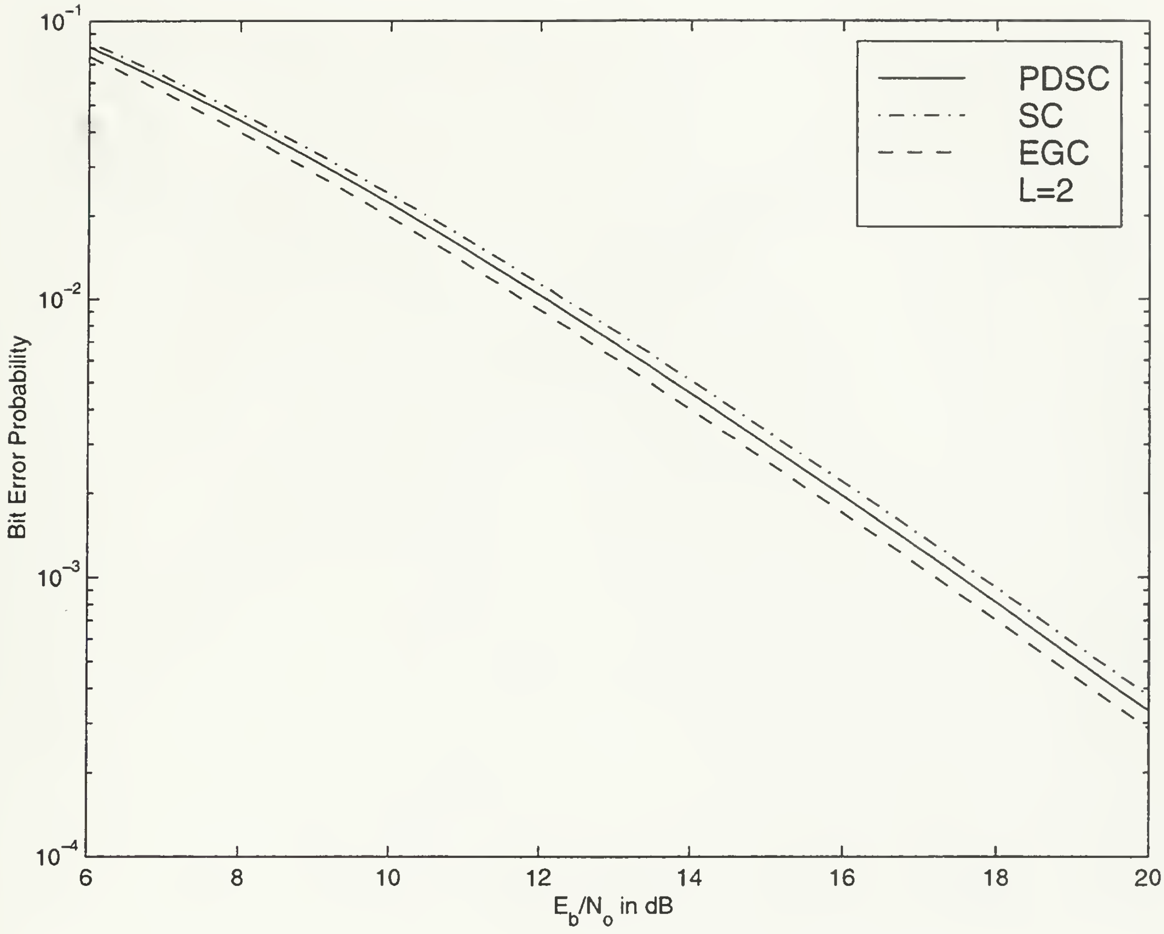


**Figure 14.** Receiver performance of PDSC-1, SC-1, and EGC over a Rayleigh fading channel for L=2.

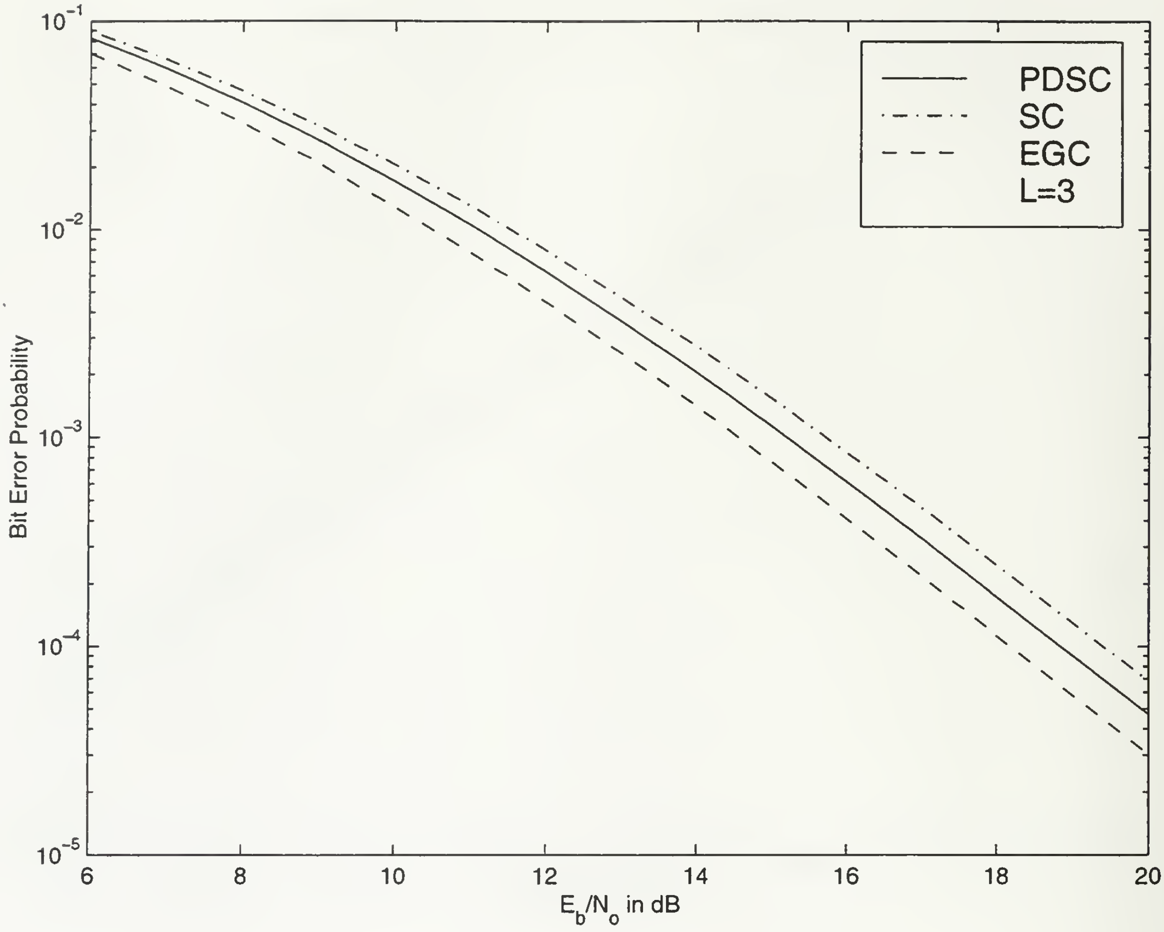


**Figure 15.** Receiver performance of PDSC-1, SC-1, and EGC over a Rayleigh fading channel for L=3.

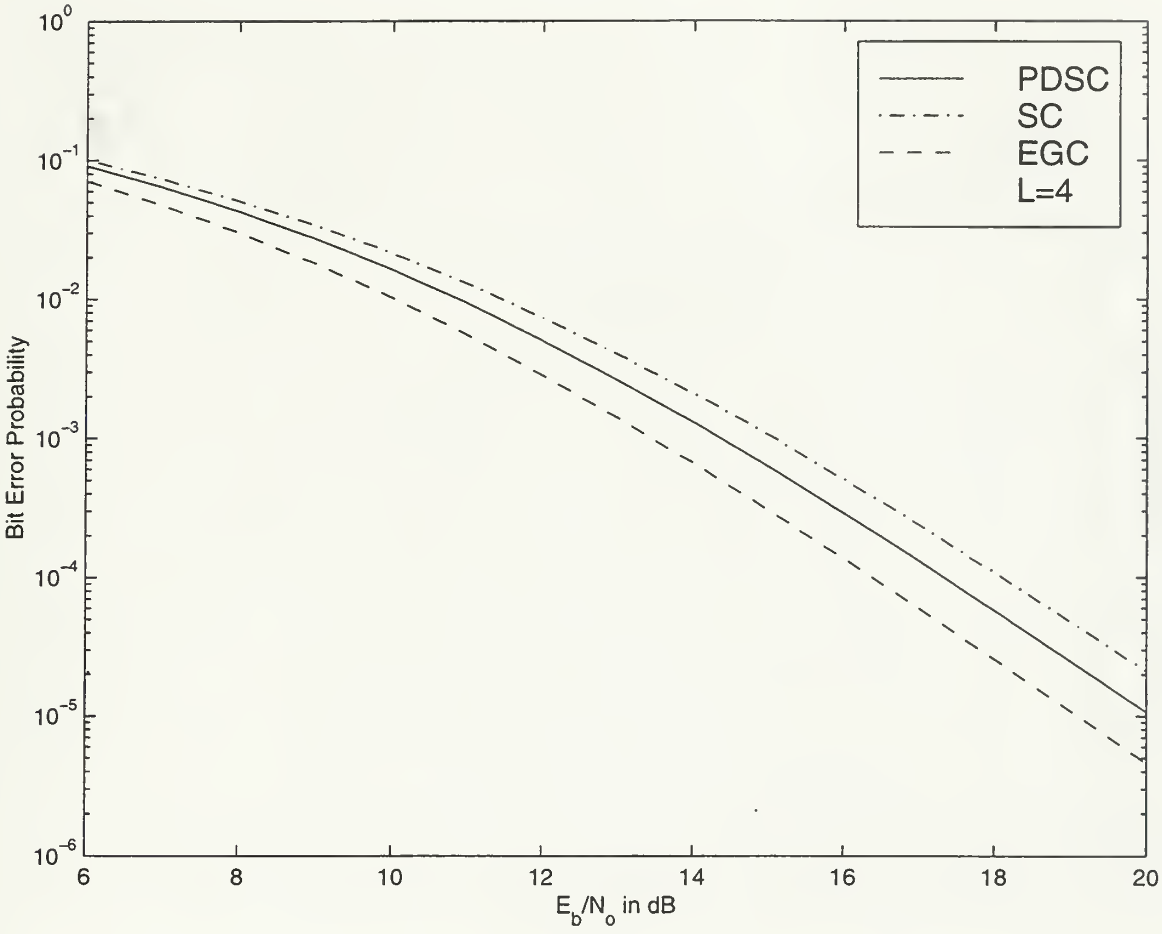


**Figure 16.** Receiver performance of PDSC-1, SC-1, and EGC over a Rayleigh fading channel for L=4.

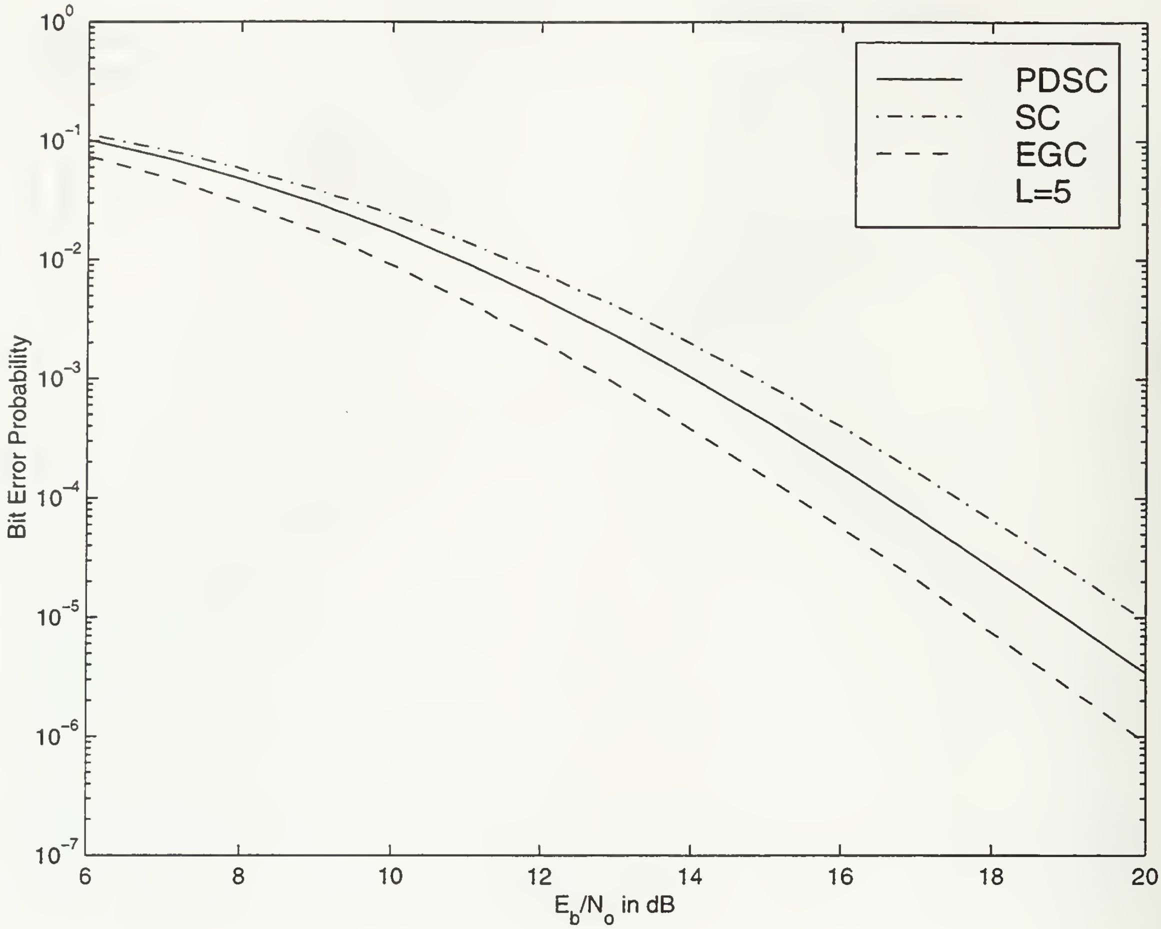


**Figure 17.** Receiver performance of PDSC-1, SC-1, and EGC over a Rayleigh fading channel for L=5.

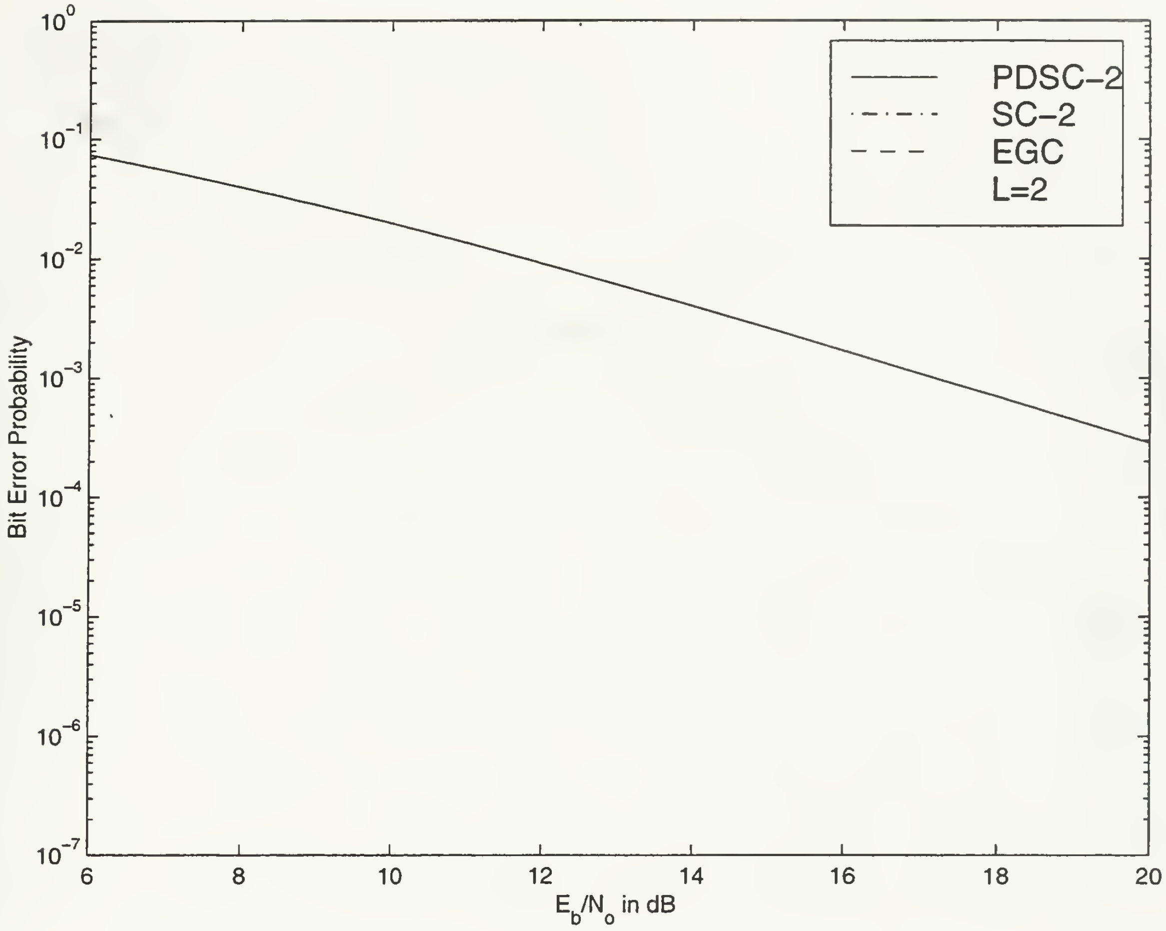


**Figure 18**. Receiver performance of PDSC-2, SC-2, and EGC over a Rayleigh fading channel for L=2.

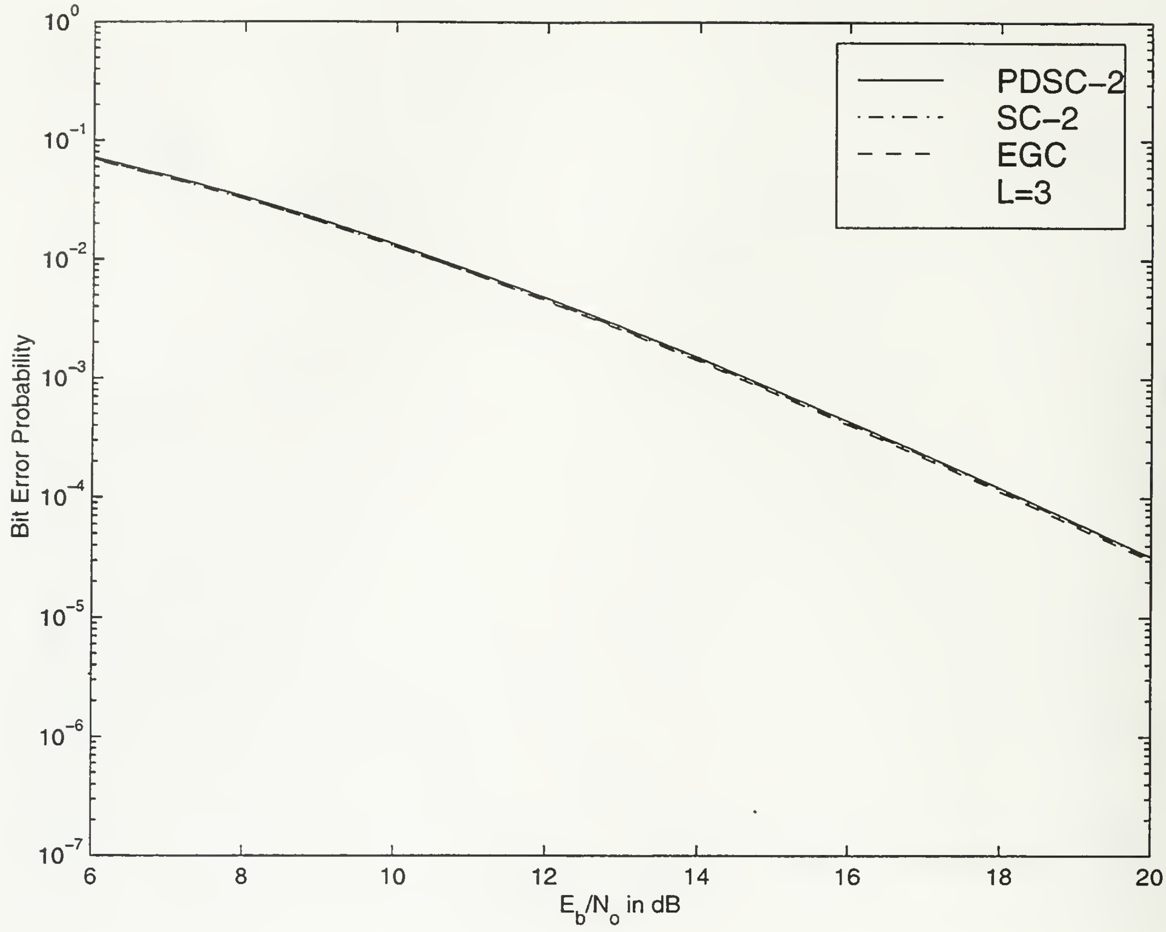


**Figure 19**. Receiver performance of PDSC-2, SC-2, and EGC over a Rayleigh fading channel for L=3.

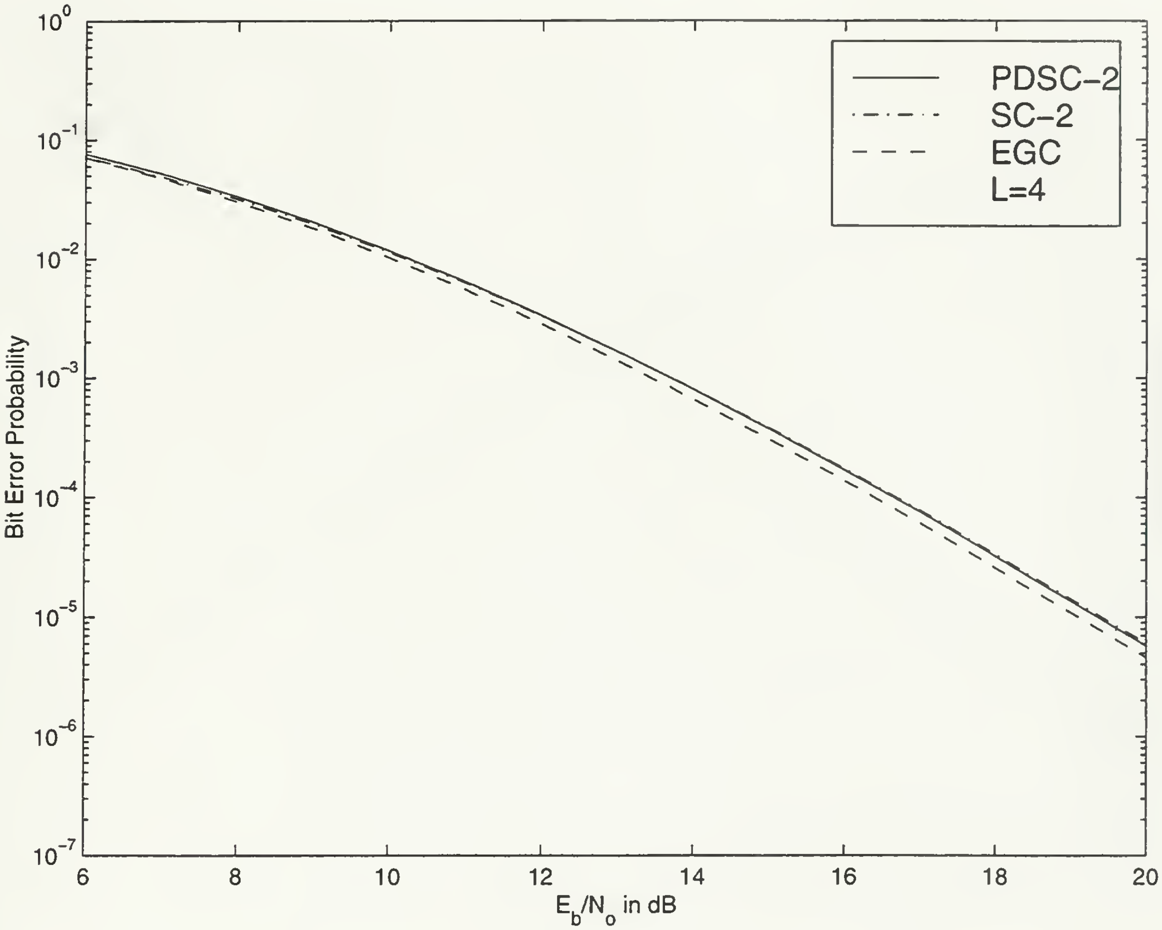


**Figure 20.** Receiver performance of PDSC-2, SC-2, and EGC over a Rayleigh fading channel for L=4.

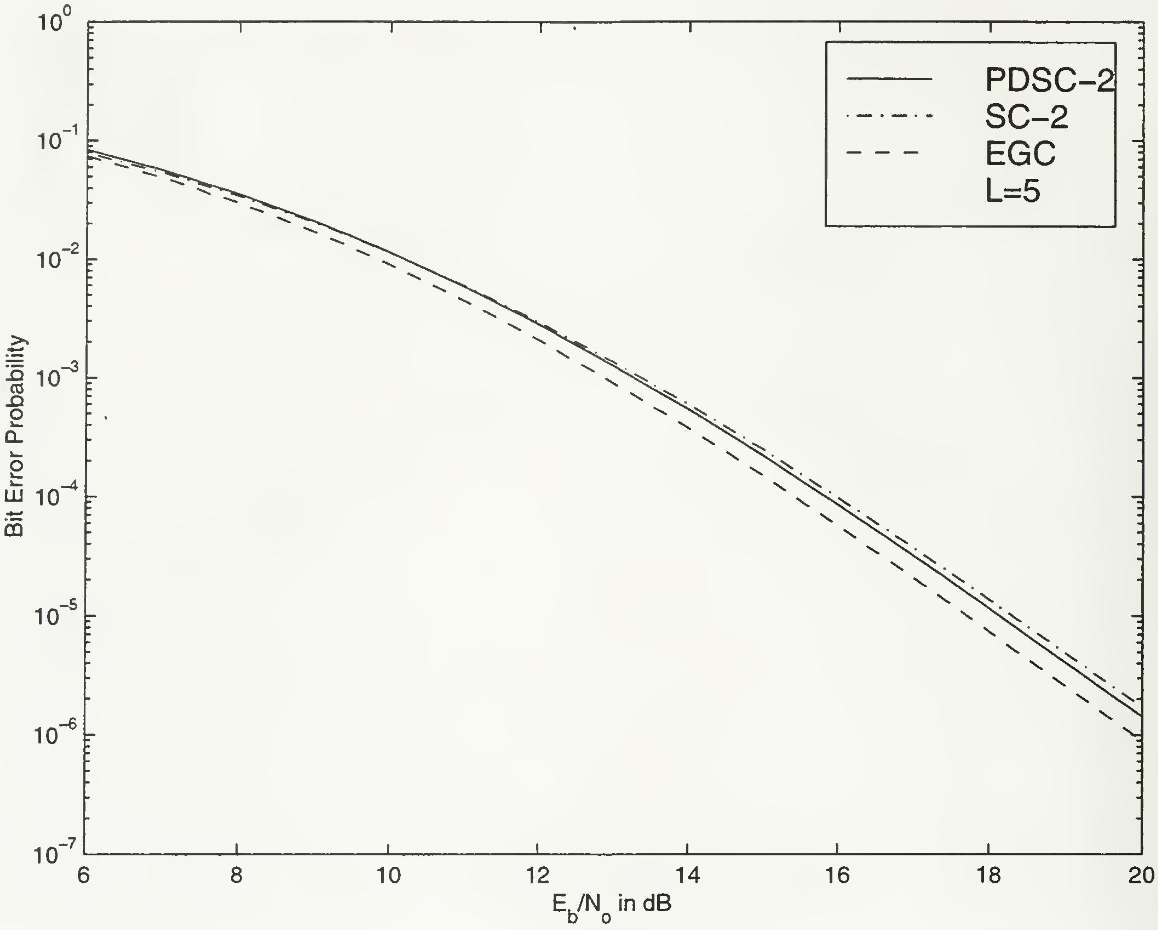


**Figure 21.** Receiver performance of PDSC-2, SC-2, and EGC over a Rayleigh fading channel for L=5.

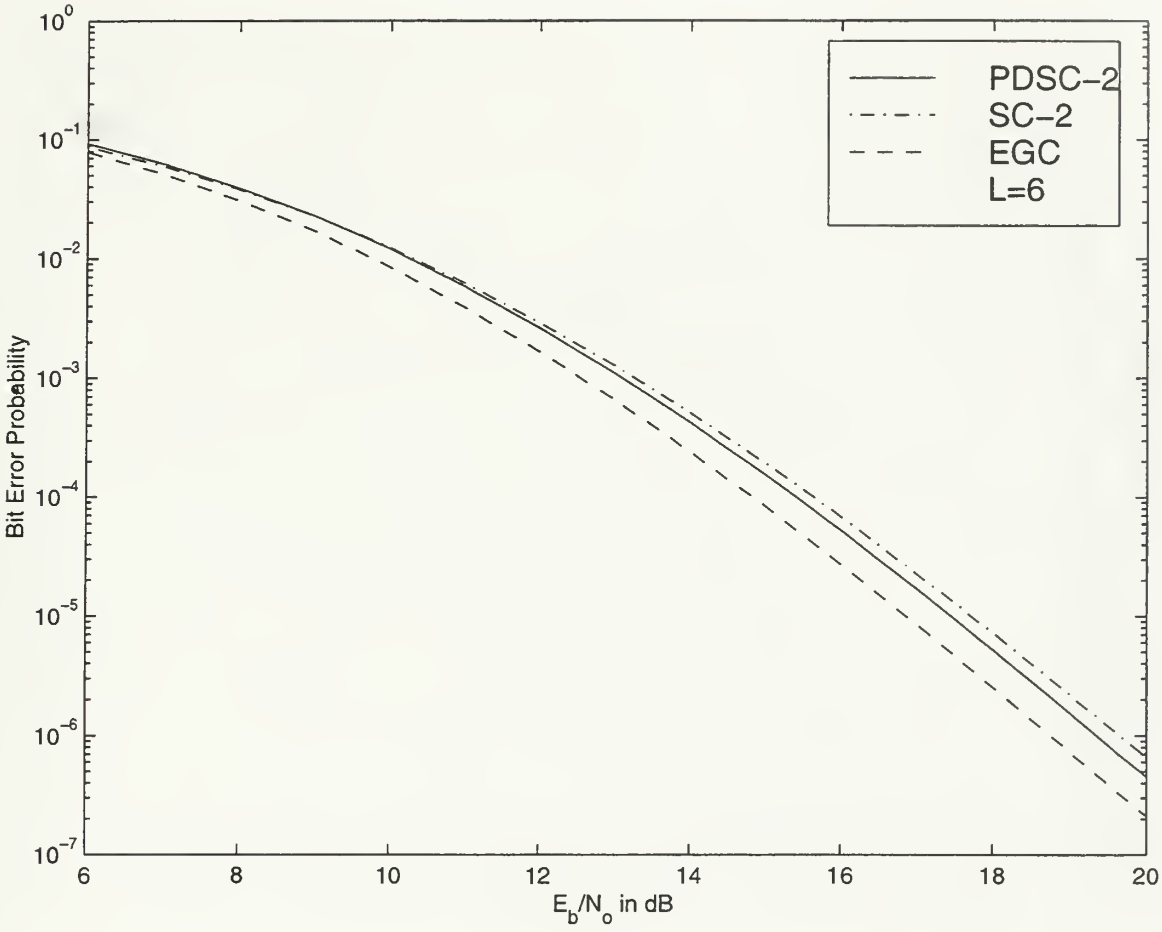


**Figure 22.** Receiver performance of PDSC-2, SC-2, and EGC over a Rayleigh fading channel for L=6.

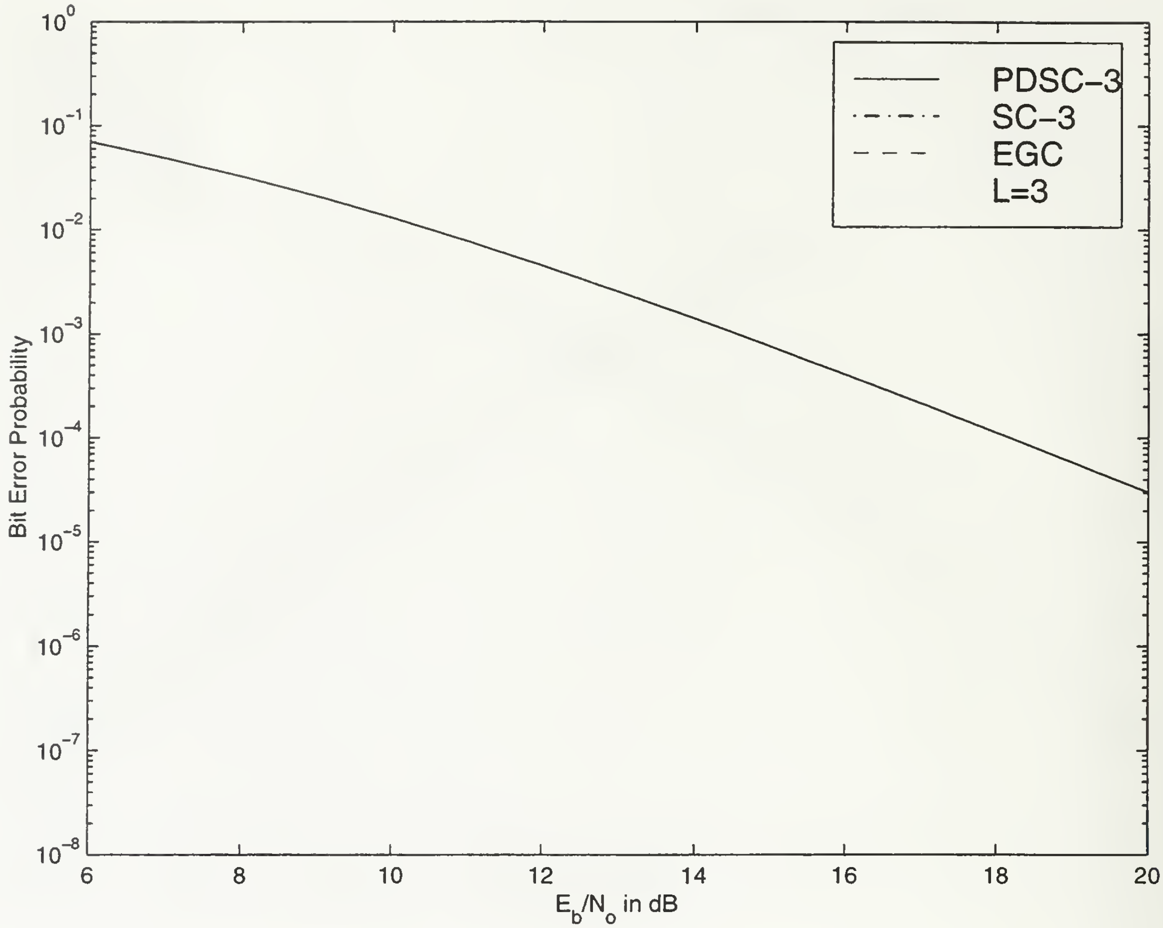


**Figure 23.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=3.

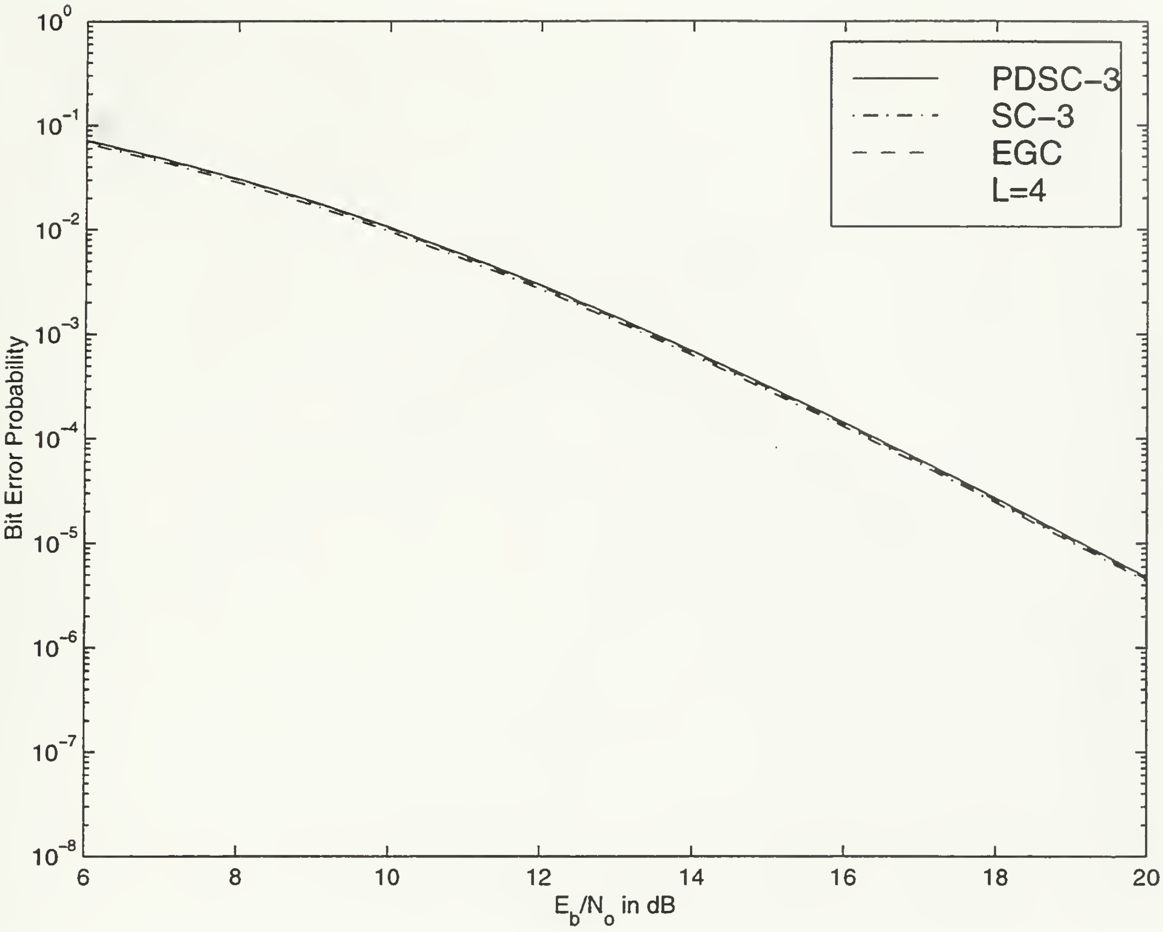


**Figure 24.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=4.

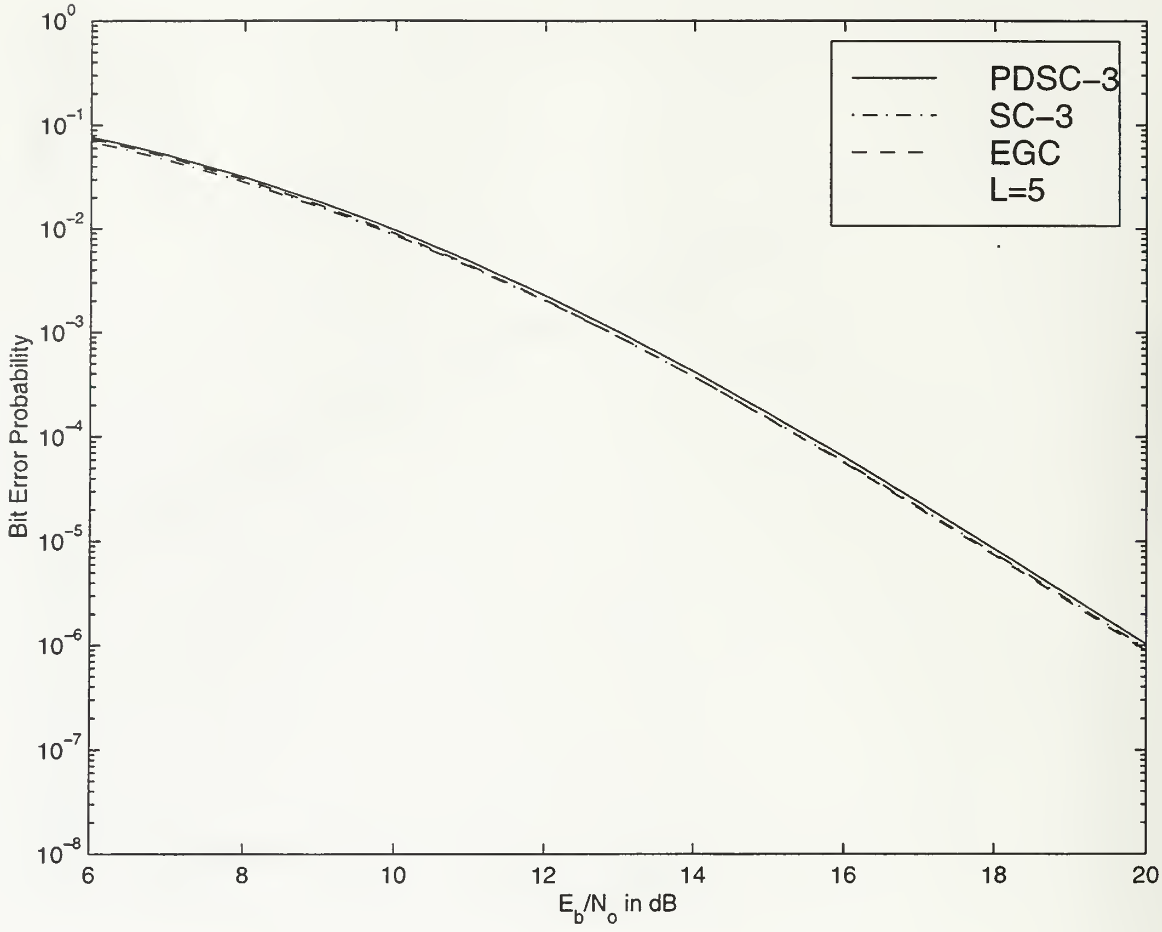


**Figure 25.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=5.

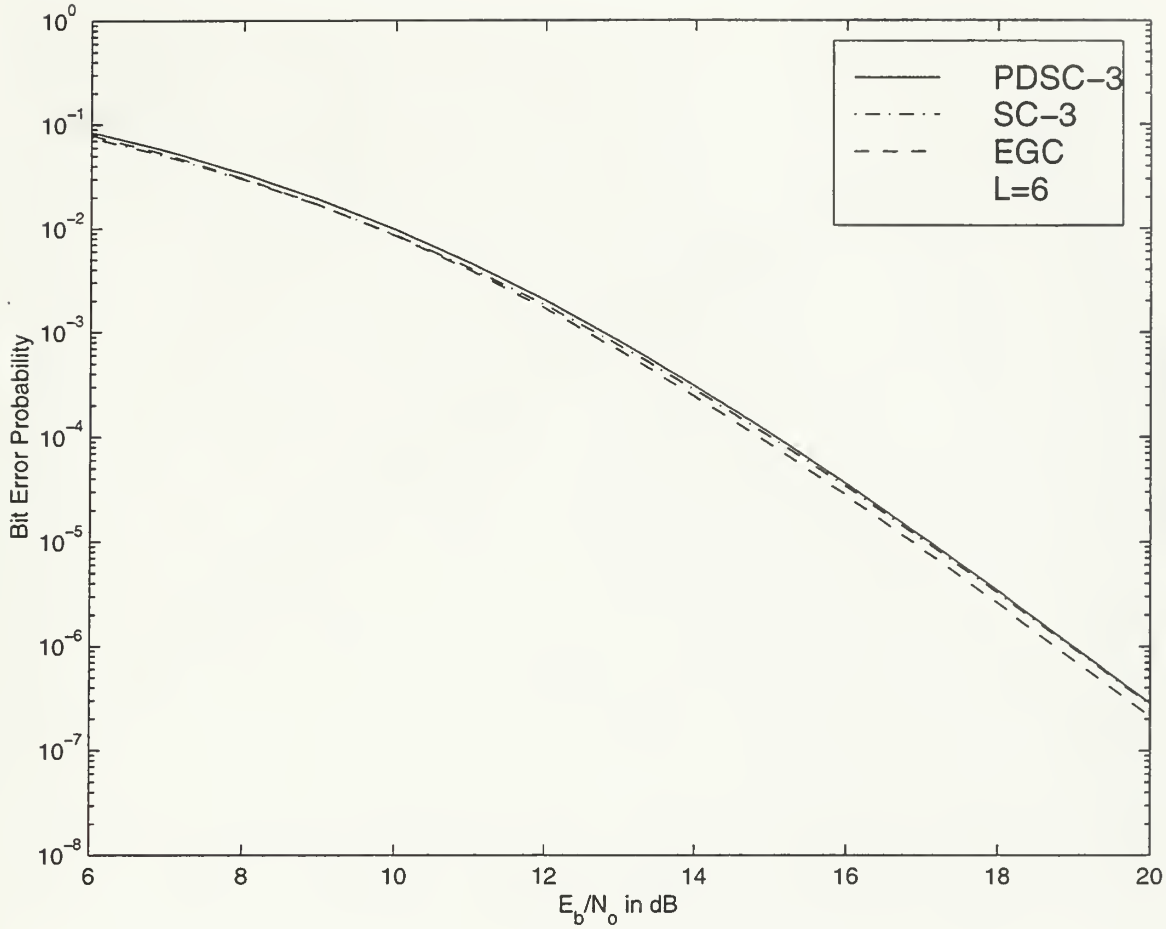


**Figure 26.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=6.

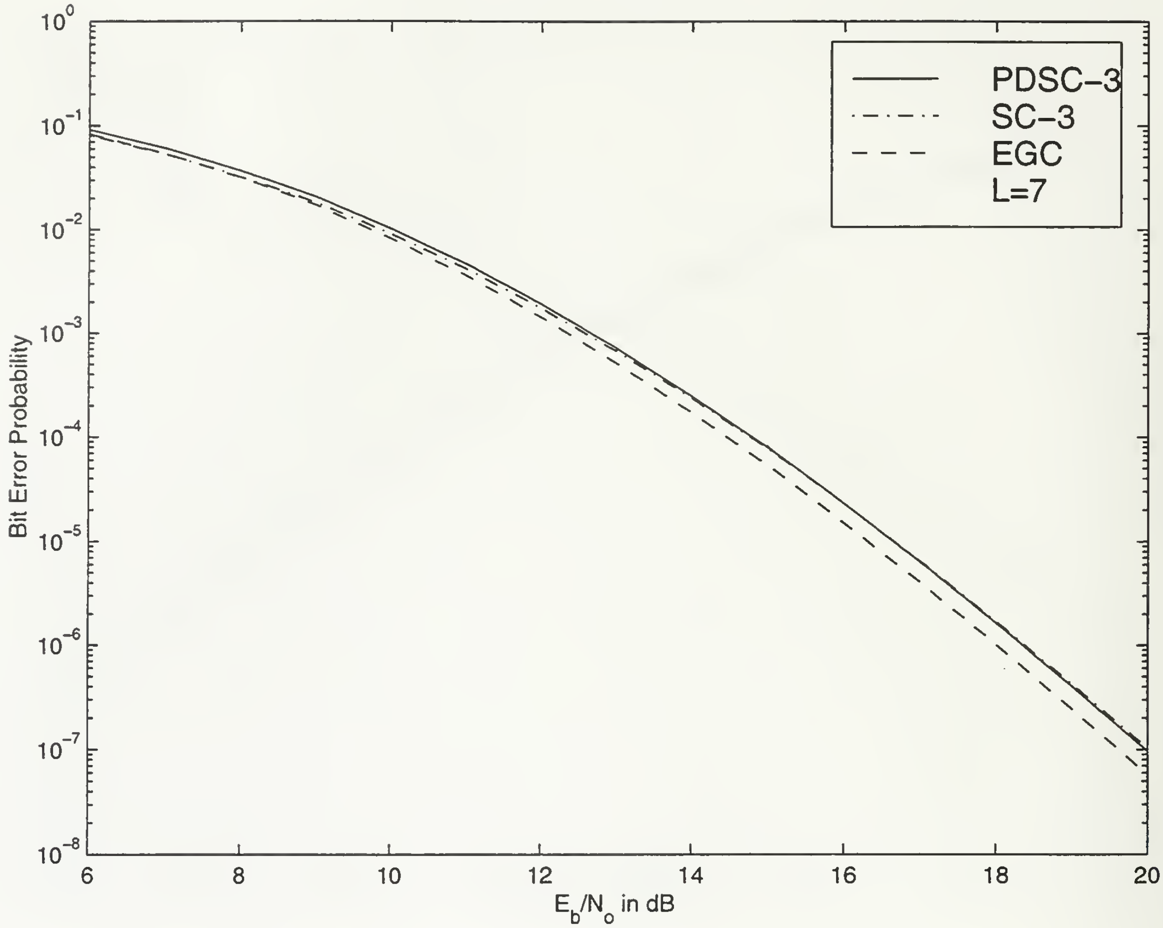


**Figure 27.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=7.

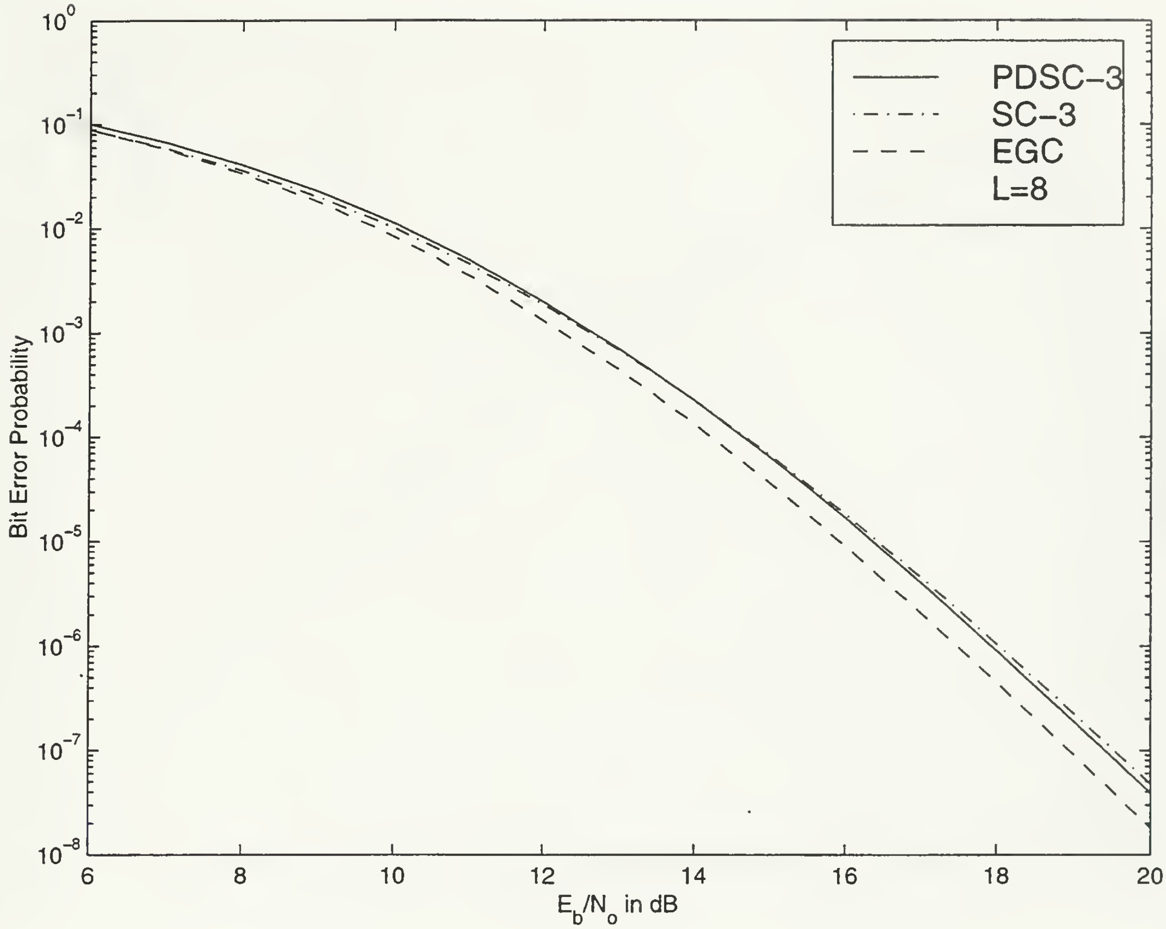


**Figure 28.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=8.

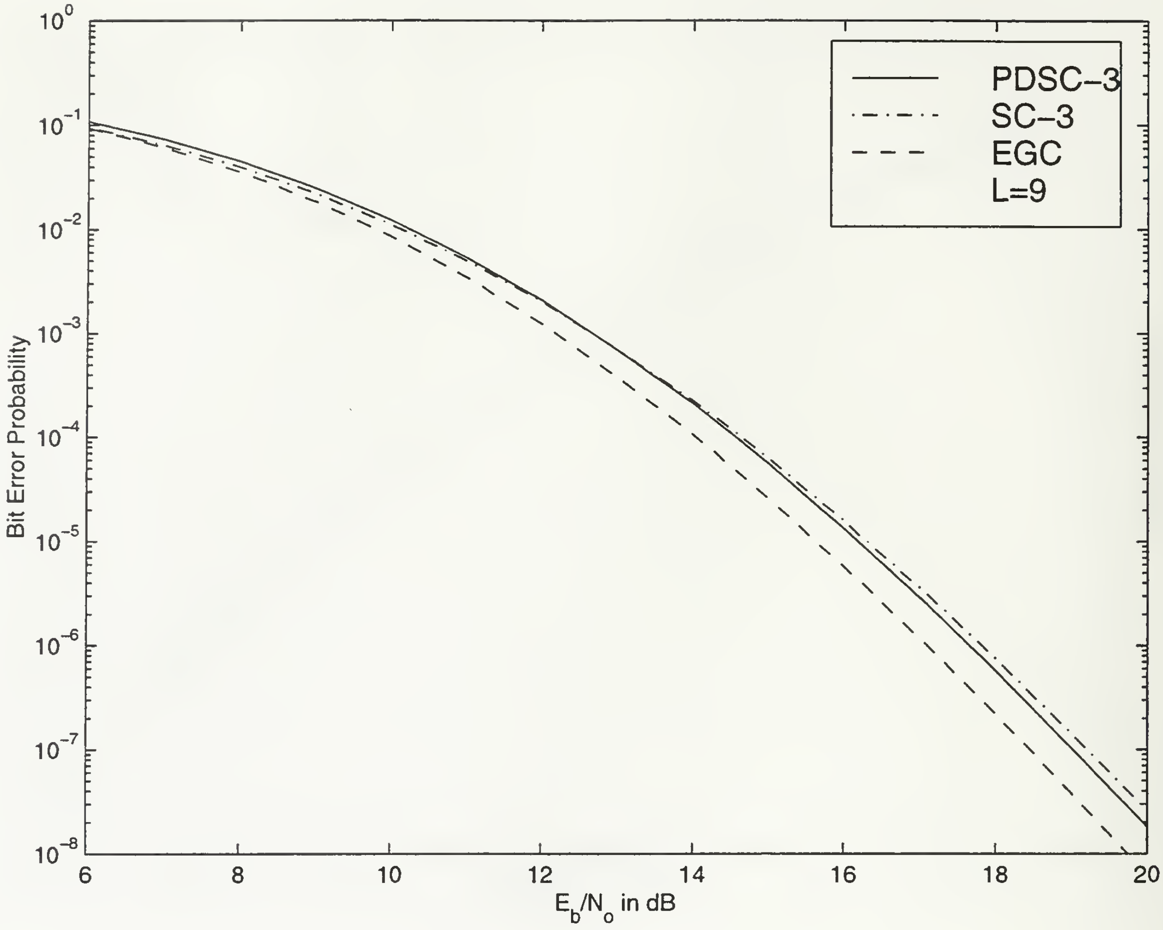


**Figure 29.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=9.

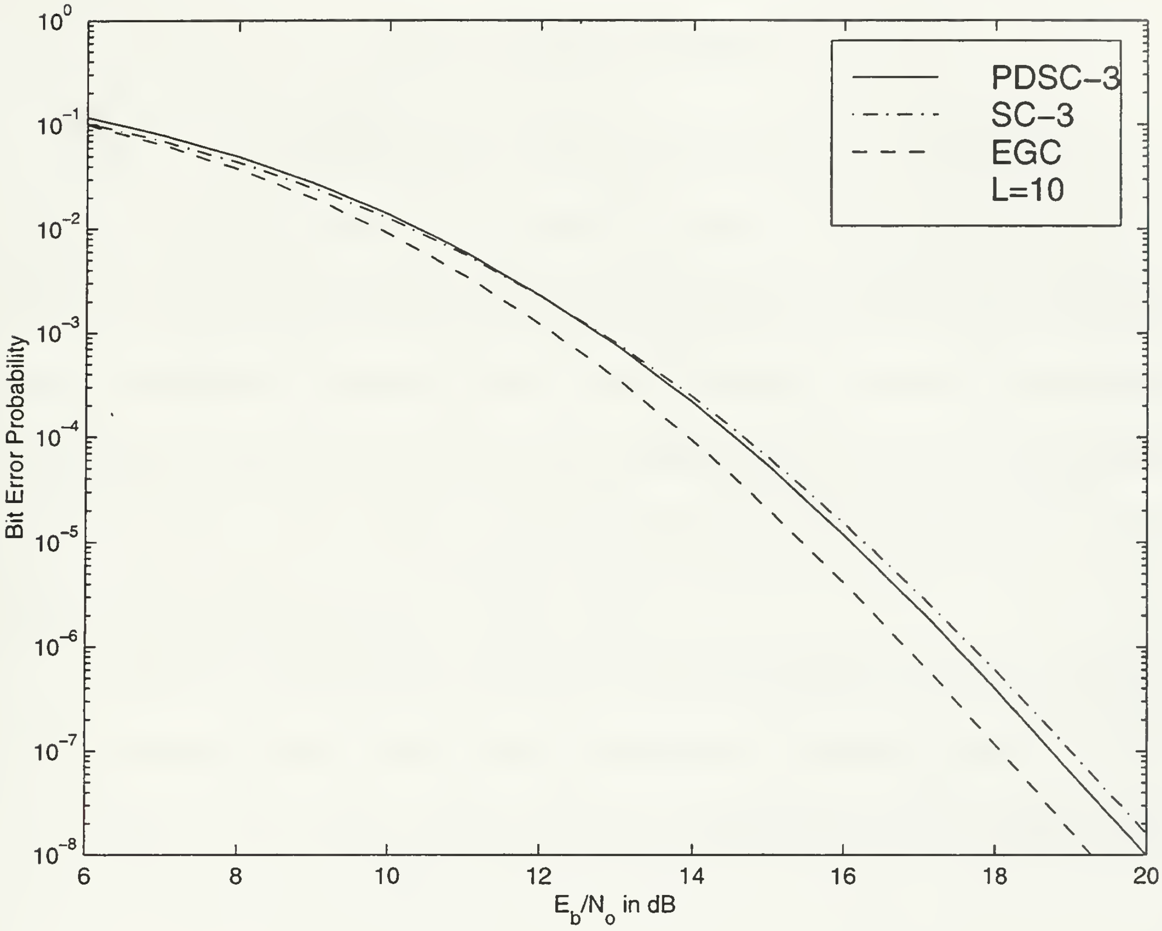


**Figure 30.** Receiver performance of PDSC-3, SC-3, and EGC over a Rayleigh fading channel for L=10.

# VII. CONCLUSIONS

The main contribution of this thesis is the analysis of the new Post-Detection Selection Combining Technique. The expressions for the bit error probability are derived for First, Second and Third order Post-Detection Selection Combining DPSK receivers. Numerical Results are compared with the results obtained from Selection Combining and Equal Gain Combining. It is observed that combining the two (or three) largest signals (PDSC-2, PDSC-3, SC-2 or SC-3 ) offers significant performance improvement over selecting the maximum signal (i.e. PDSC-1, SC-1).

First order Post-Detection Selection Combining (PDSC-1) which is done by selecting the largest signal performs better than First order Selection Combining (SC-1). As the order of the diversity increases, PDSC-1 outperforms SC-1. The improvement increases as the diversity increases.

Second order diversity combining is performed by combining the two largest signals as PDSC-2 and SC-2. Numerical results indicate that SC-2 performs the same as PDSC-2 for diversity values of L=2 to 4. However beginning L=5, PDSC-2 offers improvement over the performance of SC-2. We can conclude that as the order of diversity increases performance improvement of PDSC-2 becomes more significant relative to the SC-2. Again, PDSC-2 performs only slightly worse than the Equal Gain Combining.

In the PDSC-3 and SC-3 case, numerical results show that the SC-3 performs essentially the same as PDSC-3 for all diversity orders. Both PDSC-3 and SC-3 performances are slightly worse than EGC.

In summary, Post-Detection Selection Combining and Selection Combining receivers perform worse than EGC receivers (up to 1.5 dB in signal-to-noise ratio). On the other hand, selection combining techniques are independent of the diversity order L. This is desirable because L may vary with location, as well as time. For example, in practice different diversity channels may have different path lengths. Thus the signal-to-noise ratio of each diversity channel can be different resulting in unequal gain combining. This can cause a serious degradation for EGC receivers. SC-1 and PDSC-1, SC-2 and PDSC-2, and SC-3 and PDSC-3 have a robust performance as long as L is greater or equal to 1, 2 and 3, respectively. However, the SC method requires a more complex receiver than the EGC and PDSC methods, since it requires a pre-detction combiner.

# APPENDIX A

## DERIVATION OF THE PROBABILITY DENSITY FUNCTION FOR THE LARGEST RANDOM VARIABLE

In this appendix , the pdf expressions are derived for largest variable in a set of L i.i.d. random variables. We assume that $X_1, X_2, X_3,..., X_L$ are L independent, identically distributed random variables with probability density functions

$$f_{X_1}(x_1) = f_{X_2}(x_2) = f_{X_3}(x_3) = ... = f_{X_L}(x_L) = f_X(x). \tag{A.1}$$

An expression will be derived for the largest random variable in a set of L i.i.d. random variables as given by

$$Z = \max\{X_1, X_2, X_3, ... X_L\}. \tag{A.2}$$

In terms of order statistics, it can be interpreted as [11]

$$X_{r_1}(\xi) \le X_{r_2}(\xi) \le X_{r_3}(\xi) \le ... \le X_{r_k}(\xi) \le ... \le X_{r_L}(\xi). \tag{A.3}$$

We next form the L random variables $z_i$ such that

$$z_1 = x_{r_1} \le z_2 = x_{r_2} \le ...... \le z_k = x_{r_k} \le ..... \le z_L = x_{r_L} \tag{A.4}$$

The k-th largest random variable in this sequence is called the k-th order statistic. We determine its density function by using

$$f_Z(z)dz = \Pr\{ z < z_k \le z + dz \}. \tag{A.5}$$

This expression simply describes an event that occurs when k-1 of the random variables are less than z, and the L-k random variables are larger than z+dz. In other words, we have three subsets with associated probabilities as follows:

$$\begin{aligned} D_1 &= \{x \le z\} \longrightarrow p_1 = \Pr(D_1) = F_X(z) \\ D_2 &= \{z < x \le z + dz\} \longrightarrow p_2 = \Pr(D_2) = f_X(z)dz \\ D_3 &= \{z + dz < x\} \longrightarrow p_3 = \Pr(D_3) = 1 - F_X(z + \Delta z), \end{aligned} \tag{A.6}$$

where $F_X(x)$ is the cumulative distribution and $f_X(x)$ is the density function of x. We can define the order statistics "$D_l$ occurs $k_i$ times" as [11]

$$\frac{L!}{k_1!k_2!k_3!} p_1^{k_1} p_2^{k_2} p_3^{k_3} \quad , \quad k_1 + k_2 + k_3 = L. \tag{A.7}$$

For the event defined above, $D_1$ occurs k-1 times, $D_2$ occurs once, and $D_3$ occurs L-k times, hence

$$k_1 = k - 1 \qquad k_2 = 1 \qquad k_3 = L - k \ . \tag{A.8}$$

Substituting these expressions in (A.7), we obtain

$$f_Z(z)dy = \Pr\{z < z_k \le z + dz\} = \frac{L!}{(k-1)!(L-k)!} F_x^{k-1}(z) f_x(z) dz \left[1 - F_x(z + dz)\right]^{L-k} . \tag{A.9}$$

As a general expression , the pdf of the k-th order statistic is obtained as

$$f_k(z) = \frac{L!}{(k-1)!(L-k)!} F_x^{k-1}(z)\left[1 - F_x(z)\right]^{L-k} f_x(z) \ . \tag{A.10}$$

In this case, the largest value corresponds to k= L and the probability density function of the largest random variable in L random variables is found to be

$$f_Z(z) = L f_X(z) F_x^{L-1}(z). \tag{A.11}$$

# APPENDIX B

## DERIVATION OF THE JOINT PROBABILITY DENSITY FUNCTION FOR THE LARGEST AND THE SECOND LARGEST RANDOM VARIABLES

This time we are looking for the joint probability density function of the largest and the second largest random variable in a set of L random variables

$$Z_1 = \text{first}\max\{X_1, X_2, X_3, \ldots X_L\},$$

$$Z_2 = \text{second}\max\{X_1, X_2, X_3, \ldots X_L\}.$$

The probability density function of $z_1$ is known from the previous part as

$$f_{Z_1}(z_1) = L\, f_X(z_1) F_x^{\,L-1}(z_1). \tag{B.1}$$

For $Z_2$, $k = L-1$ and the probability density function of $Z_2$ is obtained by substituting $k = L-1$ in (A.7)

$$f_{Z_2}(z_2) = \frac{L!}{((L-1)-1)!(L-(L-1))!} F_x^{\,L-2}(z_2)[1-F_x(z_2)]^{L-(L-1)} f_x(z_2). \tag{B.2}$$

Simplifying the expression above, we obtain

$$f_{Z_2}(z_2) = L(L-1) f_X(z_2) F_x^{\,L-2}(z_2)[1-F_x(z_2)]. \tag{B.3}$$

To find the joint pdf for $Z_1$ and $Z_2$ ,we need to define a series of events as follows

$$f_{Z_1,Z_2}(z_1, z_2)dz_1 dz_2 = \Pr\{z_1 < Z_1 \le z_1 + dz_1, z_2 < Z_2 \le z_2 + dz_2\}, \tag{B.4}$$

$$
\begin{aligned}
D_1 &= \{x \le z_2\} \longrightarrow p_1 = F_x(z_2) \\
D_2 &= \{z_2 < x \le z_2 + dz_2\} \longrightarrow p_2 = f_x(z_2)dz_2 \\
D_3 &= \{z_2 + dz_2 < x < z_1\} \longrightarrow p_3 = F_x(z_1) - F_x(z_2 + dz_2) \\
D_4 &= \{z_1 < x \le z_1 + dz_1\} \longrightarrow p_4 = f_x(z_1)dz_1 \\
D_5 &= \{x > z_1 + dz_1\} \longrightarrow p_5 = 1 - F_x(z_1 + dz_1).
\end{aligned}
\tag{B.5}
$$

In this case, $D_1$ occurs (L-2) times and $k_1$= L-2 , $D_2$ occurs once and $k_2$=1, $D_3$ does not occur and $k_3$=0, $D_4$ occurs once and $k_4$=1, $D_5$ does not occur and $k_5$=0. Using

$$
\Pr(k_1, k_2, \ldots k_r) = \frac{L!}{k_1! k_2! \ldots k_r!} p_1^{k_1} p_2^{k_2} \ldots . p_3^{k_r} \quad , \quad k_1 + k_2 \ldots . + k_r = L, \tag{B.6}
$$

we obtain

$$
\begin{aligned}
f_{Z_1,Z_2}(z_1, z_2)dz_1 dz_2 &= \Pr\{z_1 < Z_1 \le z_1 + dz_1, z_2 < Z_2 \le z_2 + dz_2\} \\
&= \frac{L!}{(L-2)!} F_X^{L-2}(z_2) f_x(z_2) dz_2 f_x(z_1) dz_1 \\
&= L(L-1) f_x(z_2) f_x(z_1) F_X^{L-2}(z_2) dz_1 dz_2 .
\end{aligned}
\tag{B.7}
$$

Thus

$$
f_{Z_1,Z_2}(z_1, z_2) = L(L-1) f_x(z_1) f_x(z_2) F_X^{L-2}(z_2) . \tag{B.8}
$$

# APPENDIX C

## DERIVATION OF THE JOINT PROBABILITY DENSITY FUNCTION FOR THE FIRST, SECOND AND THIRD LARGEST RANDOM VARIABLES

In the final case, we want to find an expression for the joint pdf of the first, second and third largest random variables in a set of L random variables.

$$
\begin{aligned}
Z_1 &= \text{first max}\{X_1, X_2, X_3, \ldots X_L\}, \\
Z_2 &= \text{second max}\{X_1, X_2, X_3, \ldots X_L\}, \\
Z_3 &= \text{third max}\{X_1, X_2, X_3, \ldots X_L\}.
\end{aligned}
$$

Similar to the other cases, the joint pdf of $Z_1$ , $Z_2$ and $Z_3$ can be defined as

$$
f_{Z_1,Z_2,Z_3}(z_1, z_2, z_3) = \Pr\{z_1 < Z_1 \le z_1 + dz_1, z_2 < Z_2 \le z_2 + dz_2, z_3 < Z_3 \le z_3 + dz_3\},
$$

and in terms of sets of events as

$$
\begin{aligned}
D_1 &= \{x \le z_3\} \longrightarrow p_1 = F_x(z_3) \\
D_2 &= \{z_3 < x \le z_3 + dz_3\} \longrightarrow p_2 = f_x(z_3)dz_3 \\
D_3 &= \{z_3 + dz_3 < x < z_2\} \longrightarrow p_3 = F_x(z_2) - F_x(z_3 + dz_3) \\
D_4 &= \{z_2 < x \le z_2 + dz_2\} \longrightarrow p_4 = f_x(z_2)dz_2 \\
D_5 &= \{z_2 + dz_2 < x < z_1\} \longrightarrow p_5 = F_x(z_1) - F_x(z_2 + dz_2) \\
D_6 &= \{z_1 \langle x \le z_1 + dz_1\} \longrightarrow p_6 = f_x(z_1)dz_1 \\
D_7 &= \{x > z_1 + dz_1\} \longrightarrow p_7 = 1 - F_x(z_1 + dz_1) \quad .
\end{aligned}
\tag{C.1}
$$

In this particular case, $D_1$ occurs (L-3) times and $k_1$= L-3, $D_2$ occurs once and $k_2$=1, $D_3$ does not occur and $k_3$=0, $D_4$ occurs once and $k_4$=1, $D_5$ does not occur and $k_5$=0, $D_6$ occurs once and $k_6$=1, $D_7$ does not occur and $k_7$=0 .

Using (B.6) we obtain

$$f_{Z_1,Z_2,Z_3}(z_1, z_2, z_3) = L(L-1)(L-2)f_X(z_1)f_X(z_2)f_X(z_3)F_X^{L-3}(z_3). \tag{C.2}$$

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